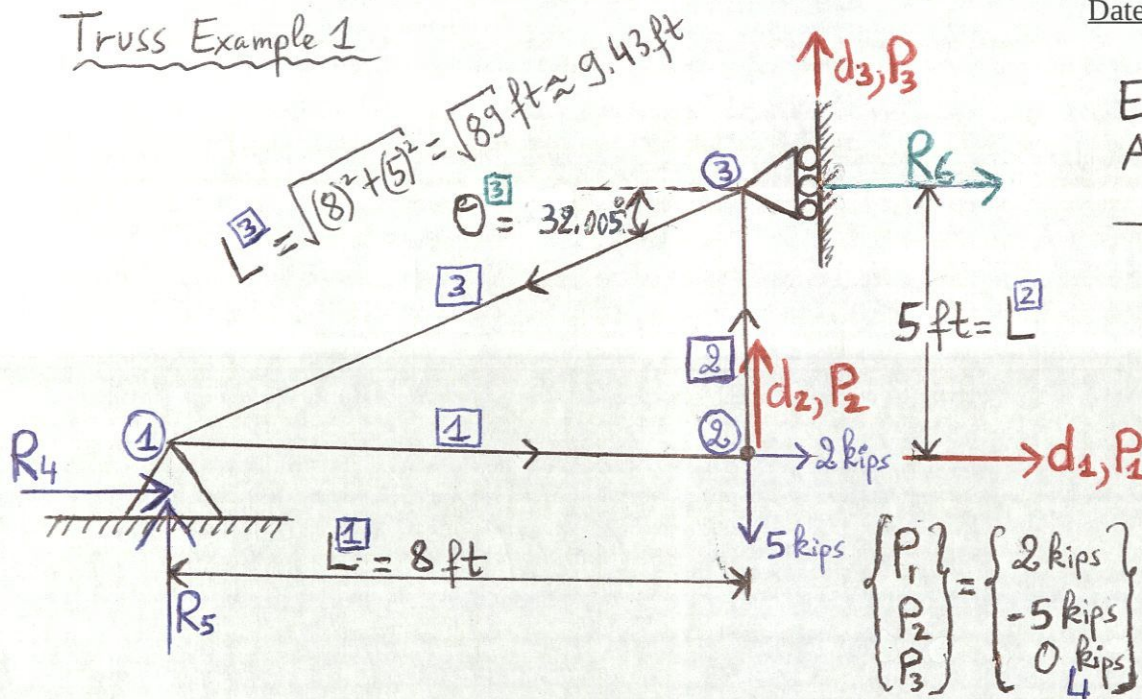




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Truss Example 1



$E = 29,000 \text{ ksi}$
 $A = 9 \text{ in}^2$

Member	Code #
1	[4 5 1 2]
2	[1 2 6 3]
3	[6 3 4 5]

$$[R]^1 = \frac{EA}{L^1} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{(29,000 \frac{\text{kips}}{\text{in}^2})(9 \text{ in}^2)}{(8 \text{ ft})(\frac{12 \text{ in}}{1 \text{ ft}})} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2,718.75 & 0 & -2,718.75 & 0 \\ 0 & 0 & 0 & 0 \\ -2,718.75 & 0 & 2,718.75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 5 \\ 1 \\ 2 \\ 4 \end{matrix} \frac{\text{kips}}{\text{in}}$$

$$[R]^2 = \frac{EA}{L^2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{(29,000 \frac{\text{kips}}{\text{in}^2})(9 \text{ in}^2)}{(5 \text{ ft})(\frac{12 \text{ in}}{1 \text{ ft}})} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4,350 & 0 & -4,350 & 0 \\ 0 & 0 & 0 & 0 \\ -4,350 & 0 & 4,350 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix} \frac{\text{kips}}{\text{in}}$$

$$[R]^3 = \frac{EA}{L^3} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{(29,000 \frac{\text{kips}}{\text{in}^2})(9 \text{ in}^2)}{(\sqrt{89} \text{ ft})(\frac{12 \text{ in}}{1 \text{ ft}})} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2,305.5 & 0 & -2,305.5 & 0 \\ 0 & 0 & 0 & 0 \\ -2,305.5 & 0 & 2,305.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 6 \\ 3 \\ 4 \\ 5 \end{matrix} \frac{\text{kips}}{\text{in}}$$

$$[K]^1 = \frac{(29,000 \text{ ksi})(9 \text{ in}^2)}{(8 \text{ ft})(\frac{12 \text{ in}}{1 \text{ ft}})} \begin{bmatrix} (\cos(\theta))^2 & (\cos(\theta)\sin(\theta)) & -(\cos(\theta))^2 & -\cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & (\sin(\theta))^2 & -\cos(\theta)\sin(\theta) & -(\sin(\theta))^2 \\ -(\cos(\theta))^2 & -\cos(\theta)\sin(\theta) & (\cos(\theta))^2 & \cos(\theta)\sin(\theta) \\ -\cos(\theta)\sin(\theta) & -(\sin(\theta))^2 & \cos(\theta)\sin(\theta) & (\sin(\theta))^2 \end{bmatrix} = [R]^1$$

$\theta^1 = 0^\circ$



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$$[K]_{2}^{2} = \frac{(29,000 \text{ ksi})(9 \text{ in}^2)}{(5 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)} \begin{bmatrix} (\cos(90))^2 & \cos(90)\sin(90) & -(\cos(90))^2 & -\cos(90)\sin(90) \\ \cos(90)\sin(90) & (\sin(90))^2 & -\cos(90)\sin(90) & -(\sin(90))^2 \\ -(\cos(90))^2 & -\cos(90)\sin(90) & (\cos(90))^2 & \cos(90)\sin(90) \\ -\cos(90)\sin(90) & -(\sin(90))^2 & \cos(90)\sin(90) & (\sin(90))^2 \end{bmatrix}$$

$\theta^2 = 90^\circ$

$$[K]_{2}^{2} = \begin{bmatrix} 1 & 2 & 6 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 4,350 & 0 & -4,350 \\ 0 & 0 & 0 & 0 \\ 0 & -4,350 & 0 & 4,350 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix} \text{ Rips in}$$

$$[K]_{3}^{3} = \frac{(29,000 \text{ ksi})(9 \text{ in}^2)}{(\sqrt{89} \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)} \begin{bmatrix} (\cos(\theta^3))^2 & \cos(\theta^3)\sin(\theta^2) & -(\cos(\theta^3))^2 & -\cos(\theta^3)\sin(\theta^2) \\ \cos(\theta^3)\sin(\theta^2) & (\sin(\theta^2))^2 & -\cos(\theta^3)\sin(\theta^2) & -(\sin(\theta^2))^2 \\ -(\cos(\theta^3))^2 & -\cos(\theta^3)\sin(\theta^2) & (\cos(\theta^3))^2 & \cos(\theta^3)\sin(\theta^2) \\ -\cos(\theta^3)\sin(\theta^2) & -(\sin(\theta^2))^2 & \cos(\theta^3)\sin(\theta^2) & (\sin(\theta^2))^2 \end{bmatrix}$$

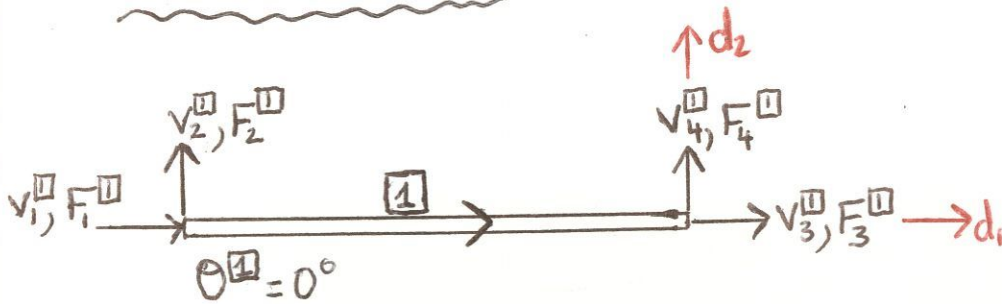
$\theta^3 = 32.005^\circ$

$$= 2,305.5 \begin{bmatrix} 0.719107 & 0.449435 & -0.719107 & -0.449435 \\ 0.449435 & 0.2808929 & -0.449435 & -0.2808929 \\ -0.719107 & -0.449435 & 0.719107 & 0.449435 \\ -0.449435 & -0.2808929 & 0.449435 & 0.2808929 \end{bmatrix}$$

$$[K]_{3}^{3} = \begin{bmatrix} 6 & 3 & 4 & 5 \\ 1,657.9 & 1,036.17 & -1,657.9 & -1,036.17 \\ 1,036.17 & 647.59858 & -1,036.17 & -647.59858 \\ -1,657.9 & -1,036.17 & 1,657.9 & 1,036.17 \\ -1,036.17 & -647.59858 & 1,036.17 & 647.59858 \end{bmatrix} \begin{matrix} 6 \\ 3 \\ 4 \\ 5 \end{matrix} \text{ Rips in}$$

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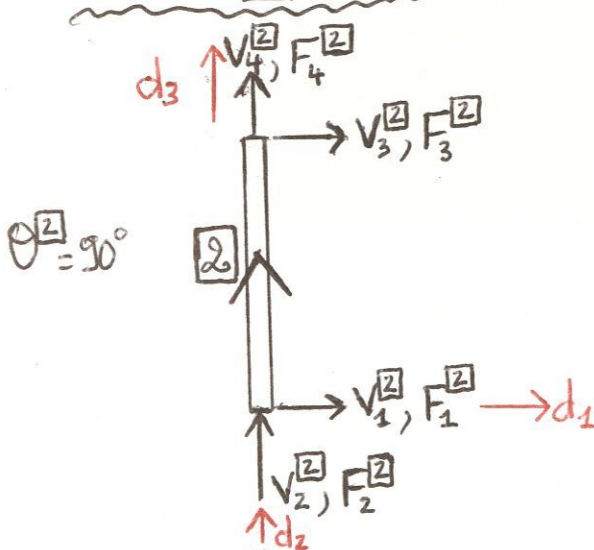
Member 1 FBD



Compatibility Conditions

$$\begin{aligned} v_1^1 &= 0 \\ v_2^1 &= 0 \\ v_3^1 &= d_1 \\ v_4^1 &= d_2 \end{aligned}$$

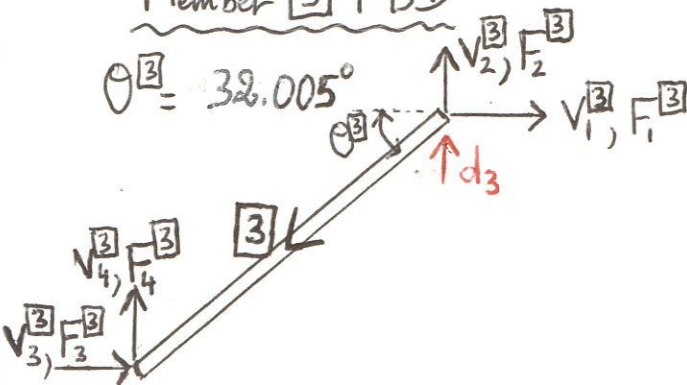
Member 2 FBD



Compatibility conditions

$$\begin{aligned} v_1^2 &= d_1 \\ v_2^2 &= d_2 \\ v_3^2 &= 0 \\ v_4^2 &= d_3 \end{aligned}$$

Member 3 FBD



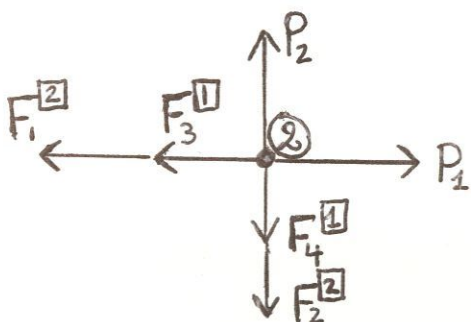
Compatibility conditions

$$\begin{aligned} v_1^3 &= 0 \\ v_2^3 &= d_3 \\ v_3^3 &= 0 \\ v_4^3 &= 0 \end{aligned}$$

Joints FBD & Assemble [S] using Rigorous Method

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Joint (2) FBD



$$\rightarrow \sum F_x = 0 ; P_1 = F_3^1 + F_1^2$$

$$\uparrow \sum F_y = 0 ; P_2 = F_4^1 + F_2^2$$

$$P_1 = \left(K_{31}^1 v_1^1 + K_{32}^1 v_2^1 + K_{33}^1 v_3^1 + K_{34}^1 v_4^1 \right) + \left(K_{11}^2 v_1^2 + K_{12}^2 v_2^2 + K_{13}^2 v_3^2 + K_{14}^2 v_4^2 \right)$$

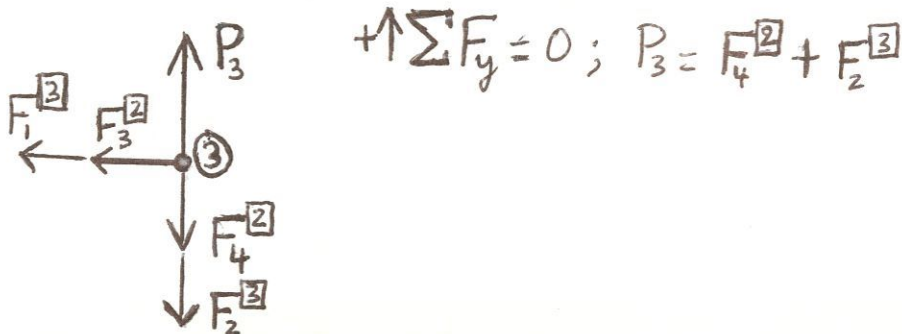
$$P_1 = \underbrace{\left(K_{33}^1 + K_{11}^2 \right)}_{S_{11}} d_1 + \underbrace{\left(K_{34}^1 + K_{12}^2 \right)}_{S_{12}} d_2 + \underbrace{\left(K_{14}^2 \right)}_{S_{13}} d_3$$

$$P_2 = \left(K_{41}^1 v_1^1 + K_{42}^1 v_2^1 + K_{43}^1 v_3^1 + K_{44}^1 v_4^1 \right) + \left(K_{21}^2 v_1^2 + K_{22}^2 v_2^2 + K_{23}^2 v_3^2 + K_{24}^2 v_4^2 \right)$$

$$P_2 = \underbrace{\left(K_{43}^1 + K_{21}^2 \right)}_{S_{21}} d_1 + \underbrace{\left(K_{44}^1 + K_{22}^2 \right)}_{S_{22}} d_2 + \underbrace{\left(K_{24}^2 \right)}_{S_{23}} d_3$$

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Joint ③ FBD



$$\begin{aligned}
 P_3 &= F_4^{[2]} + F_2^{[3]} \\
 &= \left(K_{41}^{[2]} v_1^{[2]} + K_{42}^{[2]} v_2^{[2]} + K_{43}^{[2]} v_3^{[2]} + K_{44}^{[2]} v_4^{[2]} \right) + \\
 &\quad \left(K_{21}^{[3]} v_1^{[3]} + K_{22}^{[3]} v_2^{[3]} + K_{23}^{[3]} v_3^{[3]} + K_{24}^{[3]} v_4^{[3]} \right) \\
 P_3 &= \underbrace{\left(K_{41}^{[2]} \right)}_{S_{31}} d_1 + \underbrace{\left(K_{42}^{[2]} \right)}_{S_{32}} d_2 + \underbrace{\left(K_{44}^{[2]} + K_{22}^{[3]} \right)}_{S_{33}} d_3
 \end{aligned}$$

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Assembling [S]

Using Rigorous method

$$[S] = \begin{bmatrix} K_{33}^{(1)} + K_{11}^{(2)} & K_{34}^{(1)} + K_{12}^{(2)} & K_{14}^{(2)} \\ K_{43}^{(1)} + K_{21}^{(2)} & K_{44}^{(1)} + K_{22}^{(2)} & K_{24}^{(2)} \\ K_{41}^{(2)} & K_{42}^{(2)} & K_{44}^{(2)} + K_{22}^{(3)} \end{bmatrix}$$

Using Code Number method

$$[S] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} K_{33}^{(1)} + K_{11}^{(2)} & K_{34}^{(1)} + K_{12}^{(2)} & K_{14}^{(2)} \\ K_{43}^{(1)} + K_{21}^{(2)} & K_{44}^{(1)} + K_{22}^{(2)} & K_{24}^{(2)} \\ K_{41}^{(2)} & K_{42}^{(2)} & K_{44}^{(2)} + K_{22}^{(3)} \end{bmatrix}$$

$$[S] = \begin{bmatrix} (2,718.75) + (0) & (0) + (0) & (0) \\ (0) + (0) & (0) + (4,350) & (-4,350) \\ (0) & (-4,350) & (4,350) + (647.59858) \end{bmatrix}$$

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Compute displacements

$$\{P\} = [S]\{d\}$$

$$\{d\} = [S]^{-1}\{P\}$$

$$= \begin{bmatrix} 2,718.75 & 0 & 0 \\ 0 & 4,350 & -4,350 \\ 0 & -4,350 & 4,997.598 \end{bmatrix}^{-1} \begin{Bmatrix} 2 \text{ kips} \\ -5 \text{ kips} \\ 0 \text{ kips} \end{Bmatrix}$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 0.000735632 \text{ in} \\ -0.00887026 \text{ in} \\ -0.00772083 \text{ in} \end{Bmatrix}$$

Compute Member Forces (Global coordinate system)

$$\{F\}^{\text{①}} = [K]^{\text{①}} \{v\}^{\text{①}} = \begin{bmatrix} 2,718.75 & 0 & -2,718.75 & 0 \\ 0 & 0 & 0 & 0 \\ -2,718.75 & 0 & 2,718.75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.000735632 \text{ in} \\ -0.00887026 \end{Bmatrix}$$

$$\{F\}^{\text{①}} = \begin{Bmatrix} -2 \\ 0 \\ 2 \\ 0 \end{Bmatrix} \begin{matrix} \text{4} \\ \text{Rips 5} \\ \text{1} \\ \text{2} \end{matrix}$$



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$$\{F\}^2 = [K]^2 \{w\}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4,350 & 0 & -4,350 \\ 0 & 0 & 0 & 0 \\ 0 & -4,350 & 0 & 4,350 \end{bmatrix} \begin{Bmatrix} 0.000735632 \\ -0.00887026 \\ 0 \\ -0.00772083 \end{Bmatrix}$$

$$\{F\}^2 = \begin{Bmatrix} 0 \\ -5.000002 \\ 0 \\ 5.000002 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix} \text{ kips}$$

$$\{F\}^3 = [K]^3 \{w\}^3$$

$$= \begin{bmatrix} 1,657.9 & 1,036.17 & -1,657.9 & -1,036.17 \\ 1,036.17 & 647.59858 & -1,036.17 & -647.59858 \\ -1,657.9 & -1,036.17 & 1,657.9 & 1,036.17 \\ -1,036.17 & -647.59858 & 1,036.17 & 647.59858 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.00772083 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{F\}^3 = \begin{Bmatrix} -8.000009 \\ -5 \\ 8.000009 \\ 5 \end{Bmatrix} \begin{matrix} 6 \\ 3 \\ 4 \\ 5 \end{matrix} \text{ kips}$$

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Find Reactions from {F}

$$\{F\}^{[1]} = \begin{Bmatrix} -2 \text{ kips} \\ 0 \text{ kips} \\ 2 \text{ kips} \\ 0 \text{ kips} \end{Bmatrix} \begin{matrix} 4 \\ 5 \\ 1 \\ 2 \end{matrix}$$

$$\{F\}^{[2]} = \begin{Bmatrix} 0 \text{ kips} \\ -5.00002 \text{ kips} \\ 0 \text{ kips} \\ 5.00002 \text{ kips} \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix}$$

$$\{F\}^{[3]} = \begin{Bmatrix} -8.00009 \text{ kips} \\ -5 \text{ kips} \\ 8.00009 \text{ kips} \\ 5 \text{ kips} \end{Bmatrix} \begin{matrix} 6 \\ 3 \\ 4 \\ 5 \end{matrix}$$

$$\{R\} = \begin{Bmatrix} (-2k) + (-8.00009k) \\ (0k) + (5k) \\ (0k) + (-8.00009k) \end{Bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \end{matrix}$$

Reactions

$$\{R\} = \begin{Bmatrix} R_4 \\ R_5 \\ R_6 \end{Bmatrix} = \begin{Bmatrix} 6.00009 \text{ kips} \\ 5 \text{ kips} \\ -8.00009 \text{ kips} \end{Bmatrix}$$

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Compute Transformation matrix

$$\begin{aligned}
 [T]^1 &= \begin{bmatrix} \cos(0) & \sin(0) & 0 & 0 \\ -\sin(0) & \cos(0) & 0 & 0 \\ 0 & 0 & \cos(0) & \sin(0) \\ 0 & 0 & -\sin(0) & \cos(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 (\theta^1 = 0^\circ) &
 \end{aligned}$$

$$\begin{aligned}
 [T]^2 &= \begin{bmatrix} \cos(90) & \sin(90) & 0 & 0 \\ -\sin(90) & \cos(90) & 0 & 0 \\ 0 & 0 & \cos(90) & \sin(90) \\ 0 & 0 & -\sin(90) & \cos(90) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \\
 (\theta^2 = 90^\circ) &
 \end{aligned}$$

$$\begin{aligned}
 [T]^3 &= \begin{bmatrix} \cos(\theta^3) & \sin(\theta^3) & 0 & 0 \\ -\sin(\theta^3) & \cos(\theta^3) & 0 & 0 \\ 0 & 0 & \cos(\theta^3) & \sin(\theta^3) \\ 0 & 0 & -\sin(\theta^3) & \cos(\theta^3) \end{bmatrix} \\
 (\theta^3 = -32.005^\circ) & \\
 &= \begin{bmatrix} 0.8480018487 & -0.5299932684 & 0 & 0 \\ 0.5299932684 & 0.8480018487 & 0 & 0 \\ 0 & 0 & 0.8480018487 & -0.5299932684 \\ 0 & 0 & 0.5299932684 & 0.8480018487 \end{bmatrix}
 \end{aligned}$$



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Member Forces (Local Coordinate System)

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$$\{Q\}^1 = [T]^1 \{F\}^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -2 \\ 0 \\ 2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -2 \text{ kips} \\ 0 \text{ kips} \\ 2 \text{ kips} \\ 0 \text{ kips} \end{Bmatrix} \begin{matrix} 4 \\ 5 \\ 1 \\ 2 \end{matrix}$$

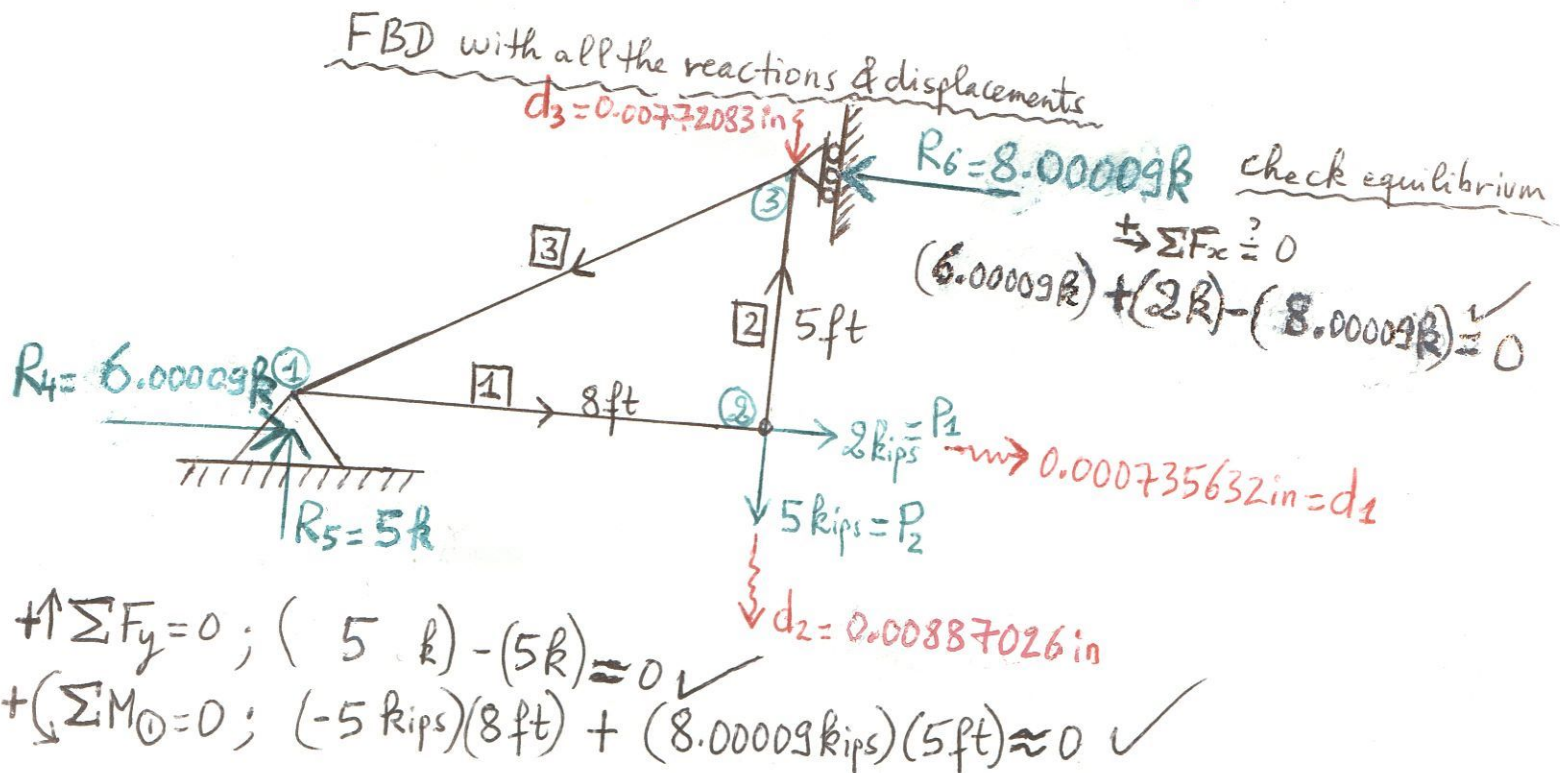
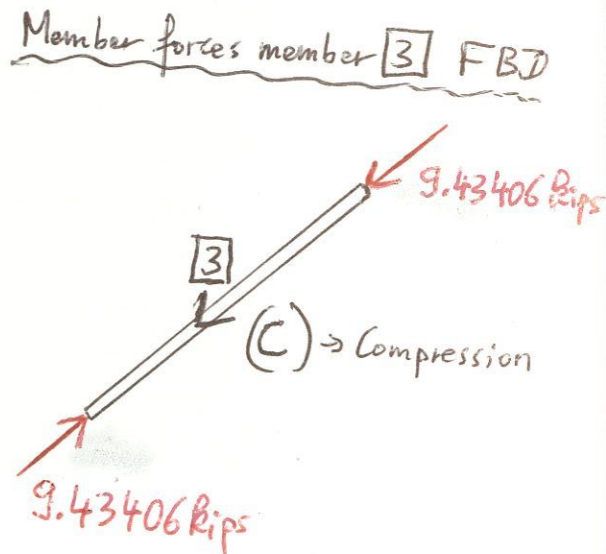
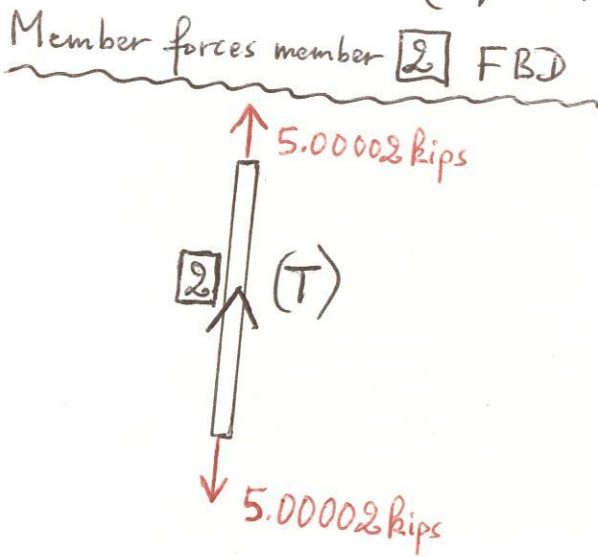
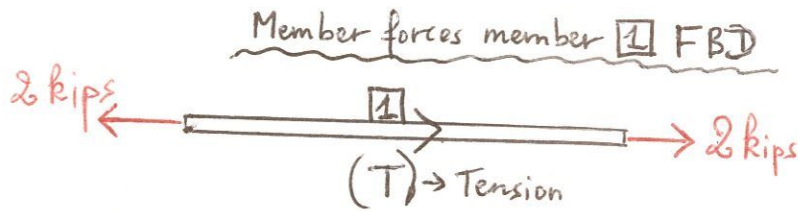
$$\{Q\}^2 = [T]^2 \{F\}^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ -5.00002 \\ 0 \\ 5.00002 \end{Bmatrix} = \begin{Bmatrix} -5.00002 \text{ kips} \\ 0 \text{ kips} \\ 5.00002 \text{ kips} \\ 0 \text{ kips} \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix}$$

$$\{Q\}^3 = [T]^3 \{F\}^3$$

$$= \begin{bmatrix} 0.8480018487 & -0.5299932684 & 0 & 0 \\ 0.5299932684 & 0.8480018487 & 0 & 0 \\ 0 & 0 & 0.8480018487 & -0.5299932684 \\ 0 & 0 & 0.5299932684 & 0.8480018487 \end{bmatrix} \begin{Bmatrix} 8.00009 \\ -5 \\ -8.00009 \\ 5 \end{Bmatrix}$$

$$\{Q\}^3 = \begin{Bmatrix} 9.43406 \\ 0.0399014 \\ -9.43406 \\ 0.0000153969 \end{Bmatrix} \begin{matrix} 6 \\ 3 \\ 4 \\ 5 \end{matrix} \text{ kips}$$

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Truss Example 1 (Structural Analysis)

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Steps

- ① Draw FBD.
- ② Label all joints in circle \circ & the members in square \square .
- ③ Label all Degrees Of Freedom (DOFs) & Reactions.
- ④ Write down the code number for each member.
- ⑤ Calculate the global stiffness matrix $[K_{ij}]$ for each member & assemble the stiffness matrix $[S_{ij}]$ either using Rigorous method (by drawing the joint FBD and writing down the equilibrium conditions of $\{P\}$ in terms of $[K]\{u\}$) or using Code number method (recommended).
- ⑥ Draw the FBD for each member & write down the compatibility conditions for each member.
- ⑦ Draw the FBD for each joint and write down the equilibrium conditions equations to find $\{P\}$ in terms of $[K]$ & $\{u\}$.
- ⑧ Assemble $\{P\}$ using Code number method.
- ⑨ Calculate the displacements, $\{d\} = \{P\}[S]^{-1}$.
- ⑩ Compute the member forces in the global coordinate system, $\{F\} = [K]\{u\}$.
- ⑪ Assemble the Reactions using Code Number method from the member forces $\{F\}$ from each member.
- ⑫ Compute Transformation matrix $[T]$ for each member.
- ⑬ Calculate the member forces in the local coordinate system, $\{Q\} = [T]\{F\}$ and indicate whether the member is in Compression (C) or Tension (T).
- ⑭ Check equilibrium in order to make sure that it is satisfied: $\sum F_x = \sum F_y = \sum M = 0$.