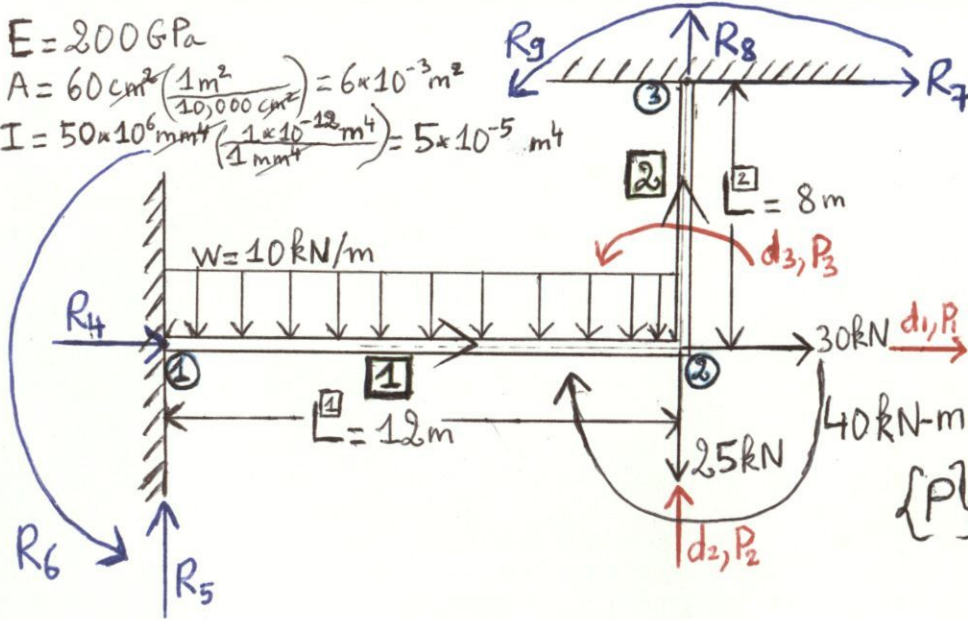


$E = 200 \text{ GPa}$
 $A = 60 \text{ cm}^2 \left(\frac{1 \text{ m}^2}{10,000 \text{ cm}^2} \right) = 6 \times 10^{-3} \text{ m}^2$
 $I = 50 \times 10^6 \text{ mm}^4 \left(\frac{1 \times 10^{-12} \text{ m}^4}{1 \text{ mm}^4} \right) = 5 \times 10^{-5} \text{ m}^4$

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Member	Code Number
1	4 5 6 1 2 3
2	1 2 3 7 8 9

Assemble $\{P\}$

$$\{P\} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 30 \text{ kN} \\ -25 \text{ kN} \\ -40 \text{ kN-m} \end{Bmatrix}$$

$$[K] = \frac{EI}{L^3} \begin{bmatrix} \frac{AL^2}{I} \cos^2(\theta) + 12 \sin^2(\theta) & \left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) & -6L \sin(\theta) & -\left(\frac{AL^2}{I} \cos^2(\theta) + 12 \sin^2(\theta)\right) \\ \left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) & \frac{AL^2}{I} \sin^2(\theta) + 12 \cos^2(\theta) & 6L \cos(\theta) & -\left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) \\ -6L \sin(\theta) & 6L \cos(\theta) & 4L^2 & 6L \sin(\theta) \\ -\left(\frac{AL^2}{I} \cos^2(\theta) + 12 \sin^2(\theta)\right) & -\left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) & 6L \sin(\theta) & \frac{AL^2}{I} \cos^2(\theta) + 12 \sin^2(\theta) \\ -\left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) & -\left(\frac{AL^2}{I} \sin^2(\theta) + 12 \cos^2(\theta)\right) & -6L \cos(\theta) & \left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) \\ -6L \sin(\theta) & 6L \cos(\theta) & 2L^2 & 6L \sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} \left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) & -6L \sin(\theta) \\ -\left(\frac{AL^2}{I} \sin^2(\theta) + 12 \cos^2(\theta)\right) & 6L \cos(\theta) \\ -6L \cos(\theta) & 2L^2 \\ \left(\frac{AL^2}{I} - 12\right) \cos(\theta) \sin(\theta) & 6L \sin(\theta) \\ \left(\frac{AL^2}{I} \sin^2(\theta) + 12 \cos^2(\theta)\right) & -6L \cos(\theta) \\ -6L \cos(\theta) & 4L^2 \end{bmatrix}$$

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$$\begin{aligned}
 [K]^{(1)} &= \begin{bmatrix} 1 \times 10^5 & 0 & 0 & -1 \times 10^5 & 0 & 0 \\ 0 & 69.44 & 416.67 & 0 & -69.44 & 416.67 \\ 0 & 416.67 & 3.33 \times 10^3 & 0 & -416.67 & 1.666 \times 10^3 \\ -1 \times 10^5 & 0 & 0 & 1 \times 10^5 & 0 & 0 \\ 0 & -69.44 & -416.67 & 0 & 69.44 & -416.67 \\ 0 & 416.67 & 1.666 \times 10^3 & 0 & -416.67 & 3.33 \times 10^3 \end{bmatrix} \frac{RN}{m} \\
 \theta^{(1)} &= 0^\circ
 \end{aligned}$$

$$\begin{aligned}
 [K]^{(2)} &= \begin{bmatrix} 234.375 & 0 & -937.5 & -234.375 & 0 & -937.5 \\ 0 & 1.5 \times 10^5 & 0 & 0 & -1.5 \times 10^5 & 0 \\ -937.5 & 0 & 5 \times 10^3 & 937.5 & 0 & 2.5 \times 10^3 \\ -234.375 & 0 & 937.5 & 234.375 & 0 & 937.5 \\ 0 & -1.5 \times 10^5 & 0 & 0 & 1.5 \times 10^5 & 0 \\ -937.5 & 0 & 2.5 \times 10^3 & 937.5 & 0 & 5 \times 10^3 \end{bmatrix} \frac{RN}{m} \\
 \theta^{(2)} &= 90^\circ
 \end{aligned}$$

$$[R]^{(1)} = [K]^{(1)}$$

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$$[K] = \frac{EI}{L^3} \begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ \frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & 6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

$$[K]^2 = \begin{bmatrix} 1.5 \times 10^5 & 0 & 0 & -1.5 \times 10^5 & 0 & 0 \\ 0 & 234.375 & 937.5 & 0 & -234.375 & 937.5 \\ 0 & 937.5 & 5 \times 10^3 & 0 & -937.5 & 2.5 \times 10^3 \\ -1.5 \times 10^5 & 0 & 0 & 1.5 \times 10^5 & 0 & 0 \\ 0 & -234.375 & 937.5 & 0 & 234.375 & -937.5 \\ 0 & 937.5 & 2.5 \times 10^3 & 0 & -937.5 & 5 \times 10^3 \end{bmatrix} \frac{RN}{m}$$

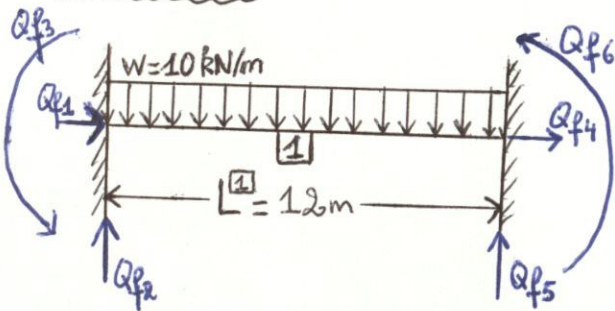
$$[T]^1_{\theta^1=0^\circ} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & 0 & 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T]^2_{\theta^2=90^\circ} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & 0 & 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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Fixed-End Forces

Member 1



$$Q_{f1} = 0$$

$$Q_{f2} = \frac{wL}{2} \left[1 - \frac{p_1^2}{L^4} (2L^3 - 2p_1^2 L - p_2^3) - \frac{p_2^3}{L^4} (2L - p_2) \right] = \frac{(10 \text{ kN/m})(12 \text{ m})}{2} = 60 \text{ kN}$$

$$Q_{f3} = \frac{wL^2}{12} \left[1 - \frac{p_1^2}{L^4} (6L^2 - 8p_1 L + 3p_1^2) - \frac{p_2^3}{L^4} (4L - 3p_2) \right] = \frac{(10 \text{ kN/m})(12 \text{ m})^2}{12} = 120 \text{ kN-m}$$

$$Q_{f4} = 0$$

$$Q_{f5} = Q_{f2} = 60 \text{ kN}$$

$$Q_{f6} = -Q_{f3} = -120 \text{ kN-m}$$

$$\{F_f\}^1 = \{Q_f\}^1 = \begin{Bmatrix} 0 \\ 60 \text{ kN} \\ 120 \text{ kN-m} \\ 0 \\ 60 \text{ kN} \\ -120 \text{ kN-m} \end{Bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{matrix}$$

$$\{F_f\}^2 = \{0\}$$

Member 2

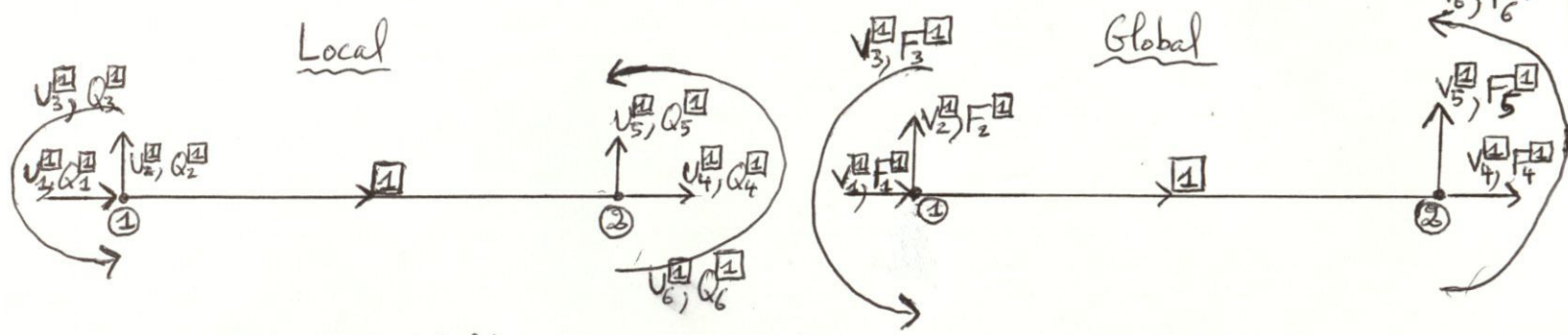
$$\{Q_f\}^2 = \{F_f\}^2 = 0 \rightarrow \text{No member forces}$$

Assemble $\{P_f\}$ using Code number method

$$\{P_f\} = \begin{Bmatrix} 0 \\ 60 \text{ kN} \\ -120 \text{ kN-m} \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

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FBD Member 1

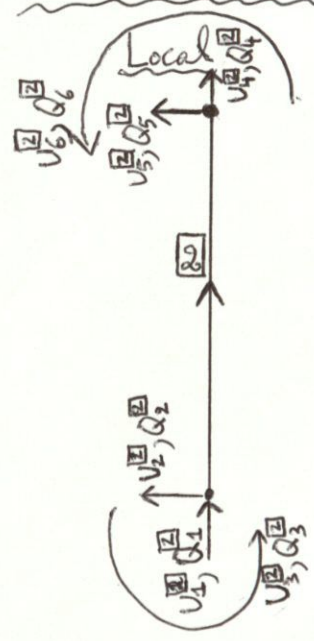


Compatibility

$$\begin{aligned} V_1^1 &= U_1^1 = 0 \\ V_2^1 &= U_2^1 = 0 \\ V_3^1 &= U_3^1 = 0 \end{aligned}$$

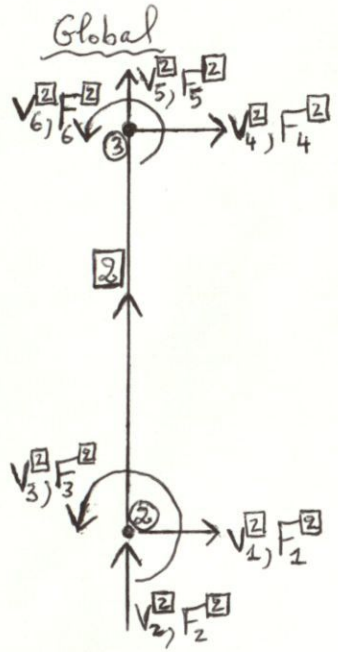
$$\begin{aligned} V_4^1 &= U_4^1 = d_1 \\ V_5^1 &= U_5^1 = d_2 \\ V_6^1 &= U_6^1 = d_3 \end{aligned}$$

FBD Member 2



⇒ Compatibility

$$\begin{aligned} U_1^2 &= d_2 \\ U_2^2 &= d_1 \\ U_3^2 &= d_3 \\ U_4^2 &= 0 \\ U_5^2 &= 0 \\ U_6^2 &= 0 \end{aligned}$$

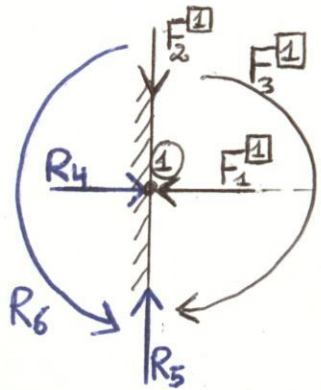


⇒ Compatibility

$$\begin{aligned} V_1^2 &= d_1 \\ V_2^2 &= d_2 \\ V_3^2 &= d_3 \\ V_4^2 &= 0 \\ V_5^2 &= 0 \\ V_6^2 &= 0 \end{aligned}$$

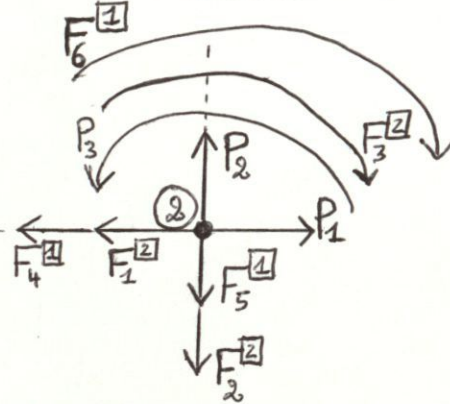
Date: 12th December 2019

Joint ① FBD



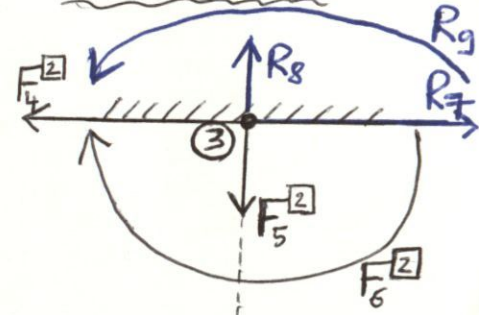
$$\begin{aligned} \rightarrow \sum F_x = 0; & \quad R_4 = F_1^1 \\ \uparrow \sum F_y = 0; & \quad R_5 = F_2^1 \\ \downarrow \sum M_{\text{①}} = 0; & \quad R_6 = F_3^1 \end{aligned}$$

Joint ② FBD



$$\begin{aligned} \rightarrow \sum F_x = 0; & \quad P_1 = F_4^1 + F_1^2 \\ \uparrow \sum F_y = 0; & \quad P_2 = F_5^1 + F_2^2 \\ \downarrow \sum M_{\text{②}} = 0; & \quad P_3 = F_6^1 + F_3^2 \end{aligned}$$

Joint ③ FBD



$$\begin{aligned} \rightarrow \sum F_x = 0; & \quad R_7 = F_4^2 \\ \uparrow \sum F_y = 0; & \quad R_8 = F_5^2 \\ \downarrow \sum M_{\text{③}} = 0; & \quad R_9 = F_6^2 \end{aligned}$$

$$P_1 = [F_4^1] + [F_1^2]$$

$$\begin{aligned} &= [(F_{f4}^1) + (K_{42}^1 v_1^1 + K_{42}^1 v_2^1 + K_{43}^1 v_3^1 + K_{44}^1 v_4^1 + K_{45}^1 v_5^1 + K_{46}^1 v_6^1)] + \\ &= [(F_{f1}^2) + (K_{11}^2 v_1^2 + K_{12}^2 v_2^2 + K_{13}^2 v_3^2 + K_{14}^2 v_4^2 + K_{15}^2 v_5^2 + K_{16}^2 v_6^2)] + \\ \{P_1\} &= \underbrace{(F_{f4}^1 + F_{f1}^2)}_{P_{f1}} + \underbrace{(K_{44}^1 + K_{11}^2)}_{S_{11}} d_1 + \underbrace{(K_{45}^1 + K_{12}^2)}_{S_{12}} d_2 + \underbrace{(K_{46}^1 + K_{13}^2)}_{S_{13}} d_3 \end{aligned}$$

$$P_2 = [F_5^1] + [F_2^2]$$

$$\begin{aligned} &= [(F_{f6}^1) + (K_{51}^1 v_1^1 + K_{52}^1 v_2^1 + K_{53}^1 v_3^1 + K_{54}^1 v_4^1 + K_{55}^1 v_5^1 + K_{56}^1 v_6^1)] + \\ &= [(F_{f2}^2) + (K_{21}^2 v_1^2 + K_{22}^2 v_2^2 + K_{23}^2 v_3^2 + K_{24}^2 v_4^2 + K_{25}^2 v_5^2 + K_{26}^2 v_6^2)] \end{aligned}$$

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$$\{P_2\} = \underbrace{(F_{f5}^{[1]} + F_{f2}^{[2]})}_{P_{f2}} + \underbrace{(K_{54}^{[1]} + K_{21}^{[2]})}_{S_{21}} d_1 + \underbrace{(K_{55}^{[1]} + K_{22}^{[2]})}_{S_{22}} d_2 + \underbrace{(K_{56}^{[1]} + K_{23}^{[2]})}_{S_{23}} d_3$$

$$P_2 = [F_6^{[1]}] + [F_3^{[2]}]$$

$$= \left[(F_{f6}^{[1]}) + (K_{61}^{[1]} \cancel{v_1^0} + K_{62}^{[1]} \cancel{v_2^0} + K_{63}^{[1]} \cancel{v_3^0} + K_{64}^{[1]} \cancel{v_4^0} + K_{65}^{[1]} \cancel{v_5^0} + K_{66}^{[1]} \cancel{v_6^0}) + \right. \\ \left. (F_{f3}^{[2]}) + (K_{31}^{[2]} \cancel{v_1^{d1}} + K_{32}^{[2]} \cancel{v_2^{d2}} + K_{33}^{[2]} \cancel{v_3^{d3}} + K_{34}^{[2]} \cancel{v_4^0} + K_{35}^{[2]} \cancel{v_5^0} + K_{36}^{[2]} \cancel{v_6^0}) \right]$$

$$\{P_3\} = \underbrace{(F_{f6}^{[1]} + F_{f3}^{[2]})}_{P_{f3}} + \underbrace{(K_{64}^{[1]} + K_{31}^{[2]})}_{S_{31}} d_1 + \underbrace{(K_{65}^{[1]} + K_{32}^{[2]})}_{S_{32}} d_2 + \underbrace{(K_{66}^{[1]} + K_{33}^{[2]})}_{S_{33}} d_3$$

Assembling [S] using Code Number Method

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$$[K]^1 = \begin{bmatrix} 4 & 5 & 6 & 1 & 2 & 3 \\ 1 \times 10^5 & 0 & 0 & -1 \times 10^5 & 0 & 0 \\ 0 & 69.44 & 416.67 & 0 & -69.44 & 416.67 \\ 0 & 416.67 & 3.33 \times 10^3 & 0 & -416.67 & 1.666 \times 10^3 \\ -1 \times 10^5 & 0 & 0 & 1 \times 10^5 & 0 & 0 \\ 0 & -69.44 & -416.67 & 0 & 69.44 & -416.67 \\ 0 & 416.67 & 1.666 \times 10^3 & 0 & -416.67 & 3.33 \times 10^3 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{matrix} \frac{RN}{m}$$

$$[K]^2 = \begin{bmatrix} 1 & 2 & 3 & 7 & 8 & 9 \\ 234.375 & 0 & -937.5 & -234.375 & 0 & -937.5 \\ 0 & 1.5 \times 10^5 & 0 & 0 & -1.5 \times 10^5 & 0 \\ -937.5 & 0 & 5 \times 10^3 & 937.5 & 0 & 2.5 \times 10^3 \\ -234.375 & 0 & 937.5 & 234.375 & 0 & 937.5 \\ 0 & -1.5 \times 10^5 & 0 & 0 & 1.5 \times 10^5 & 0 \\ -937.5 & 0 & 2.5 \times 10^3 & 937.5 & 0 & 5 \times 10^3 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{matrix} \frac{RN}{m}$$

$$[S] = \begin{bmatrix} (1 \times 10^5) + (234.375) & (0) + (0) & (0) + (-937.5) \\ = 1.00234375 \times 10^5 & = 0 & = -937.5 \\ (0) + (0) & (69.44) + (1.5 \times 10^5) & (-416.67) + (0) \\ = 0 & = 1.5006944 \times 10^5 & = -416.67 \\ (0) + (-937.5) & (-416.67) + (0) & (3.33 \times 10^3) + (5 \times 10^3) \\ = -937.5 & = -416.67 & = 8.33 \times 10^3 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \frac{RN}{m}$$

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$$\{P\} = \{P_f\} + [S]\{d\}$$

Solve for displacements

$$\{d\} = [\{P\} - \{P_f\}] \times [S]^{-1}$$

$$= \begin{bmatrix} 30 \\ -25 \\ -40 \end{bmatrix} - \begin{bmatrix} 0 \\ 60 \\ -120 \end{bmatrix} * \begin{bmatrix} 1.00234375 \times 10^5 & 0 & -937.5 \\ 0 & 1.5006944 \times 10^5 & -416.67 \\ -937.5 & -416.67 & 8.33 \times 10^3 \end{bmatrix}^{-1}$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 0.000389281 \text{ m} \left(\frac{1,000 \text{ mm}}{1 \text{ m}} \right) = 0.389281 \text{ mm} \\ -0.000539693 \text{ m} \left(\frac{1,000 \text{ mm}}{1 \text{ m}} \right) = -0.539693 \text{ mm} \\ 0.00962066 \text{ rad} \end{Bmatrix}$$

Member Forces and Reactions

$$\{F\}^{\text{[1]}} = \{F_f\}^{\text{[1]}} + [K]^{\text{[1]}} \{V\}^{\text{[1]}}$$

$$= \begin{Bmatrix} 0 \\ 60 \text{ kN} \\ 120 \text{ kN-m} \\ 0 \\ 60 \text{ kN} \\ -120 \text{ kN-m} \end{Bmatrix} + \begin{bmatrix} 1 \times 10^5 & 0 & 0 & -1 \times 10^5 & 0 & 0 \\ 0 & 69.44 & 416.67 & 0 & -69.44 & 416.67 \\ 0 & 416.67 & 3.33 \times 10^3 & 0 & -416.67 & 1.666 \times 10^3 \\ -1 \times 10^5 & 0 & 0 & 1 \times 10^5 & 0 & 0 \\ 0 & -69.44 & -416.67 & 0 & 69.44 & -416.67 \\ 0 & 416.67 & 1.666 \times 10^3 & 0 & -416.67 & 3.33 \times 10^3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.00038 \text{ m} \\ -0.000539 \text{ m} \\ 0.00962 \text{ rad} \end{Bmatrix}$$

$$\{F\}^{\text{[1]}} = \{Q\}^{\text{[1]}} = \begin{matrix} 4 & \rightarrow R_4 \\ 5 & \rightarrow R_5 \\ 6 & \rightarrow R_6 \\ 1 & \\ 2 & \\ 3 & \end{matrix} \begin{Bmatrix} -38.9281 \text{ kN} \\ 64.046116684 \text{ kN} \\ 136.25289344 \text{ kN-m} \\ 58.39215 \text{ kN} \\ 55.95388332 \text{ kN} \\ -87.73832832 \text{ kN-m} \end{Bmatrix}$$

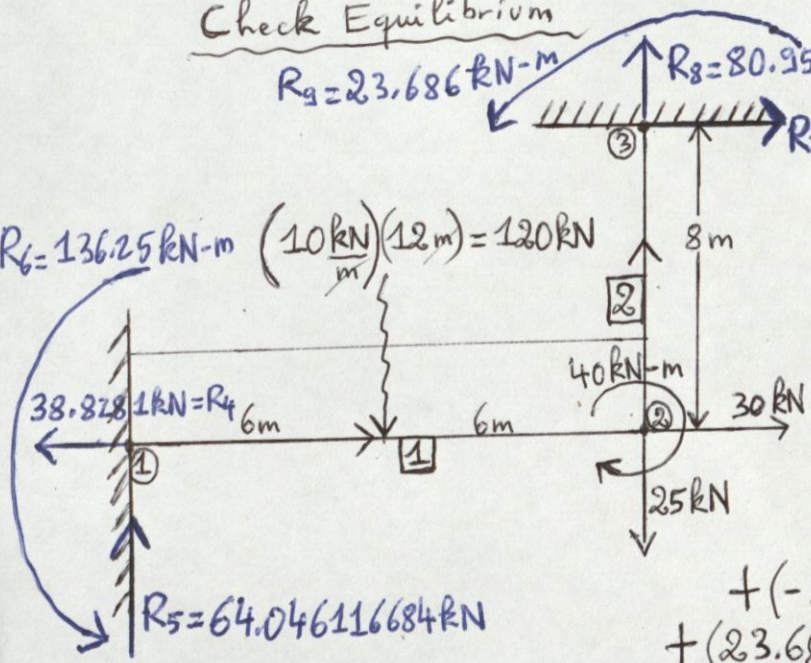
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$$\{F\}^2 = \{F\}^2 \rightarrow 0 + [K]^2 \{V\}^2$$

$$= \begin{bmatrix} 234.375 & 0 & -937.5 & -234.375 & 0 & -937.5 \\ 0 & 1.5 \times 10^5 & 0 & 0 & -1.5 \times 10^5 & 0 \\ -937.5 & 0 & 5 \times 10^3 & 937.5 & 0 & 2.5 \times 10^3 \\ -234.375 & 0 & 937.5 & 234.375 & 0 & 937.5 \\ 0 & -1.5 \times 10^5 & 0 & 0 & 1.5 \times 10^5 & 0 \\ -937.5 & 0 & 2.5 \times 10^3 & 937.5 & 0 & 5 \times 10^3 \end{bmatrix} \begin{Bmatrix} 0.000389281 \text{ m} \\ -0.000539693 \text{ m} \\ 0.00962066 \text{ rad} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{F\}^2 = \begin{Bmatrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{Bmatrix} \begin{Bmatrix} -8.928131016 \text{ kN} \\ -80.95395 \text{ kN} \\ 47.73834906 \text{ kN-m} \\ 8.928131016 \text{ kN} \rightarrow R_7 \\ 80.95395 \text{ kN} \rightarrow R_8 \\ 23.68669906 \text{ kN-m} \rightarrow R_9 \end{Bmatrix}$$

Check Equilibrium



$$\begin{aligned} + \sum F_x = 0? \\ (-38.9281 \text{ kN}) + (30 \text{ kN}) + (8.9281 \text{ kN}) \\ = 0 \checkmark \end{aligned}$$

$$\begin{aligned} + \sum F_y = 0? \\ (64.046116684 \text{ kN}) - (25 \text{ kN}) + (80.95395 \text{ kN}) \\ - (120 \text{ kN}) = 6.6684 \times 10^{-5} \approx 0 \checkmark \end{aligned}$$

$$\begin{aligned} + (\sum M_1 = 0?) \\ (-120 \text{ kN})(6 \text{ m}) + (-25 \text{ kN})(12 \text{ m}) - (40 \text{ kN-m}) \\ + (-8.928131016 \text{ kN})(8 \text{ m}) + (80.95395 \text{ kN})(12 \text{ m}) \\ + (23.68669906 \text{ kN-m}) + (136.25289344 \text{ kN-m}) = -0.038064088 \approx 0 \checkmark \end{aligned}$$