DTY
TUTORING, inc.

DTY Tutoring, inc.
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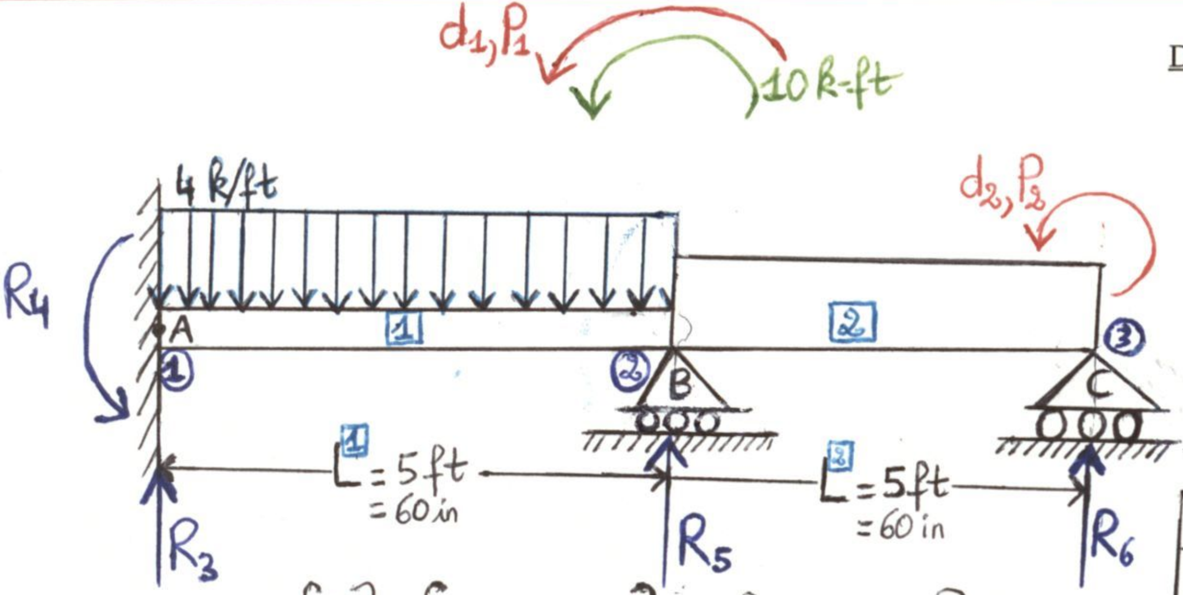
Date: 26th July 2019



Given:

$E = 29,000 \text{ ksi}$

$I^{\boxed{1}} = 200 \text{ in}^4$

$I^{\boxed{2}} = 300 \text{ in}^4$

Member	Code #
$\boxed{1}$	[3 4 5 1]
$\boxed{2}$	[5 1 6 2]

$$\{P\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{Bmatrix} 10 \text{ kips-ft} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 120 \text{ kips-in} \\ 0 \end{Bmatrix}$$

$$[k]^{\boxed{1}} = \frac{EI^{\boxed{1}}}{(L^{\boxed{1}})^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$= \frac{\left(29,000 \frac{kips}{in^2}\right)(200 in^4)}{\left[5 \cancel{ft} \left(\frac{12 in}{1 \cancel{ft}}\right)\right]^3} \begin{bmatrix} 12 & 6(60in) & -12 & 6(60in) \\ 6(60in) & 4(60in)^2 & -6(60in) & 2(60in)^2 \\ -12 & -6(60in) & 12 & -6(60in) \\ 6(60in) & 2(60in)^2 & -6(60in) & 4(60in)^2 \end{bmatrix}$$

$$[k]^{\boxed{1}} = \begin{array}{cccc|c} 3 & 4 & 5 & 1 & \\ \hline 322.22 & 9,666.67 & -322.22 & 9,666.67 & 3 \\ 9,666.67 & 386,666.67 & -9,666.67 & 193,333.33 & 4 \\ -322.22 & -9,666.67 & 322.22 & -9,666.67 & 5 \\ 9,666.67 & 193,333.33 & -9,666.67 & 386,666.67 & 1 \end{array} \quad \frac{kips}{in}$$

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$$[k]^{\boxed{2}} = \frac{EI^{\boxed{2}}}{\left[L^{\boxed{2}}\right]^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$= \frac{\left(29{,}000\,\frac{kips}{in^2}\right)\left(300\,in^4\right)}{\left(60\,in\right)^3}\begin{bmatrix} 12 & 6(60\,in) & -12 & 6(60\,in) \\ 6(60\,in) & 4(60\,in)^2 & -6(60\,in) & 2(60\,in)^2 \\ -12 & -6(60\,in) & 12 & -6(60\,in) \\ 6(60\,in) & 2(60\,in)^2 & -6(60\,in) & 4(60\,in)^2 \end{bmatrix}$$

$$[k]^{\boxed{2}} = \begin{array}{c} \begin{array}{cccc} 5 & 1 & 6 & 2 \end{array} \\ \left[\begin{array}{cccc} 483.33 & 14{,}500 & -483.33 & 14{,}500 \\ 14{,}500 & 580{,}000 & -14{,}500 & 290{,}000 \\ -483.33 & -14{,}500 & 483.33 & -14{,}500 \\ 14{,}500 & 290{,}000 & -14{,}500 & 580{,}000 \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 1 \\ 6 \\ 2 \end{array} \quad \frac{kips}{in}$$

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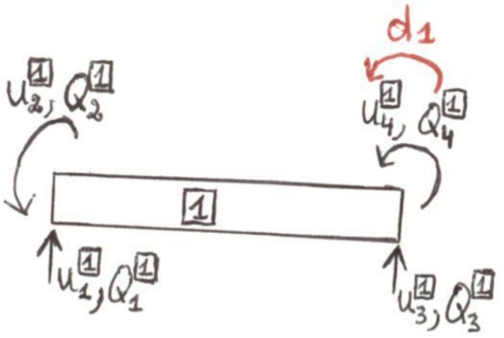
Member $\boxed{1}$ FBD



Compatibility conditions

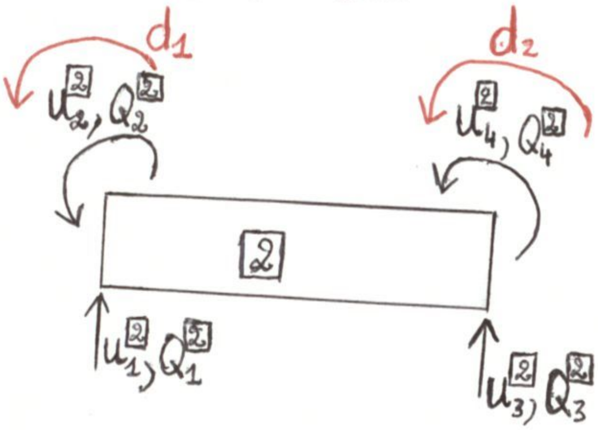
$u_1^{\boxed{1}} = 0$

$u_2^{\boxed{1}} = 0$

$u_3^{\boxed{1}} = 0$

$u_4^{\boxed{1}} = d_1$

Member $\boxed{2}$ FBD



Compatibility Conditions

$u_1^{\boxed{2}} = 0$

$u_2^{\boxed{2}} = d_1$

$u_3^{\boxed{2}} = 0$

$u_4^{\boxed{2}} = d_2$

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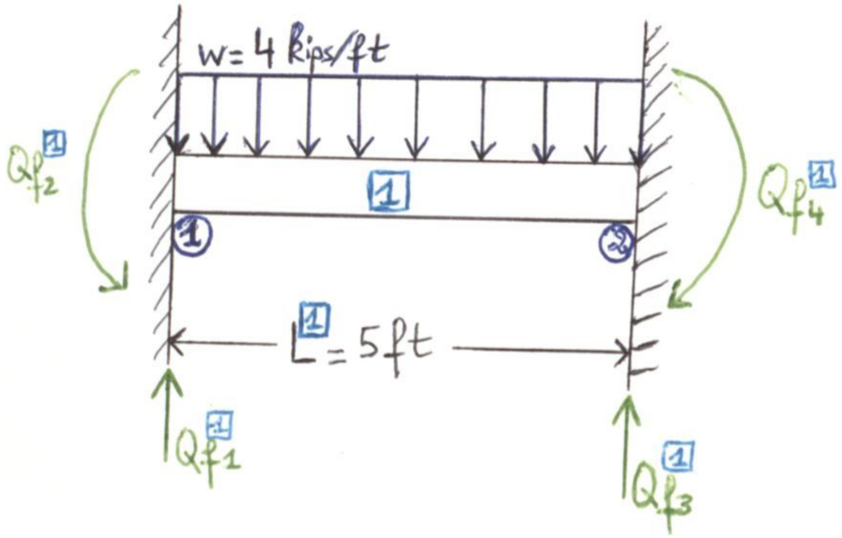
Fixed-End Forces

Member $\boxed{1}$



$$\{Q_f\}^{\boxed{1}} = \begin{Bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \end{Bmatrix}^{\boxed{1}} = \begin{Bmatrix} \frac{wL^{\boxed{1}}}{2} = \frac{(4\text{ kips/ft})(5\text{ft})}{2} = 10\text{ kips} \\ \frac{w(L^{\boxed{1}})^2}{12} = \frac{(4\text{ kips/ft})(5\text{ft})^2}{12} = 8.33\text{ kips-ft} \\ \frac{wL^{\boxed{1}}}{2} = \frac{(4\text{ kips/ft})(5\text{ft})}{2} = 10\text{ kips} \\ \frac{w(L^{\boxed{1}})^2}{12} = \frac{(4\text{ kips/ft})(5\text{ft})^2}{12} = -8.33\text{ kips-ft} \end{Bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 1 \end{matrix}$$

Member $\boxed{2}$

No fixed-end forces → $\{Q_f\}^{\boxed{2}} = \{0\} = \begin{Bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \end{Bmatrix}^{\boxed{2}} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} 5 \\ 1 \\ 6 \\ 2 \end{matrix}$

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Assembling $\{P_f\}$ using Code Number method

$$\{Q_f\}^{\boxed{1}} = \begin{Bmatrix} Q_{f_1}^{\boxed{1}} \\ Q_{f_2}^{\boxed{1}} \\ Q_{f_3}^{\boxed{1}} \\ Q_{f_4}^{\boxed{1}} \end{Bmatrix} = \left\{ \begin{array}{l} 10 \text{ kips} \\ 8.33 \text{ kips-ft} \\ 10 \text{ kips} \\ -8.33 \text{ kips-ft} \end{array} \right\} \begin{array}{l} 3 \\ 4 \\ 5 \\ 1 \end{array}$$

$$\{Q_f\}^{\boxed{2}} = \begin{Bmatrix} Q_{f_1}^{\boxed{2}} \\ Q_{f_2}^{\boxed{2}} \\ Q_{f_3}^{\boxed{2}} \\ Q_{f_4}^{\boxed{2}} \end{Bmatrix} = \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} \right\} \begin{array}{l} 5 \\ 1 \\ 6 \\ 2 \end{array}$$

$$\{P_f\} = \begin{Bmatrix} P_{f_1} \\ P_{f_2} \end{Bmatrix} = \left\{ \begin{array}{c} (-8.33) + 0 = -8.33 \text{ kips-ft} \\ 0 \end{array} \right\} \begin{array}{l} 1 \\ 2 \end{array}$$

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Assembling Stiffness Matrix [S] using Code Number Method

$$[k]^{\boxed{1}} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc|c} 3 & 4 & 5 & 1 & \\ \hline 322.22 & 9,666.67 & -322.22 & 9,666.67 & 3 \\ 9,666.67 & 386,666.67 & -9,666.67 & 193,333.33 & 4 \\ -322.22 & -9,666.67 & 322.22 & -9,666.67 & 5 \\ 9,666.67 & 193,333.33 & -9,666.67 & \underbrace{386,666.67}_{k_{44}^{\boxed{1}}} & 1 \end{array} \quad \frac{kips}{in}$$

$$[k]^{\boxed{2}} = \begin{array}{cccc|c} 5 & 1 & 6 & 2 & \\ \hline 483.33 & 14,500 & -483.33 & 14,500 & 5 \\ 14,500 & \underbrace{580,000}_{k_{22}^{\boxed{2}}} & -14,500 & \underbrace{290,000}_{k_{24}^{\boxed{2}}} & 1 \\ -483.33 & -14,500 & 483.33 & -14,500 & 6 \\ 14,500 & \underbrace{290,000}_{k_{42}^{\boxed{2}}} & -14,500 & \underbrace{580,000}_{k_{44}^{\boxed{2}}} & 2 \end{array} \quad \frac{kips}{in}$$

$$[S] = \begin{array}{cc|c} 1 & 2 & \\ \hline (580,000)+(386,666.67) = 966,666.67 & 290,000 & 1 \\ 290,000 & 580,000 & 2 \end{array} \quad \frac{kips}{in}$$

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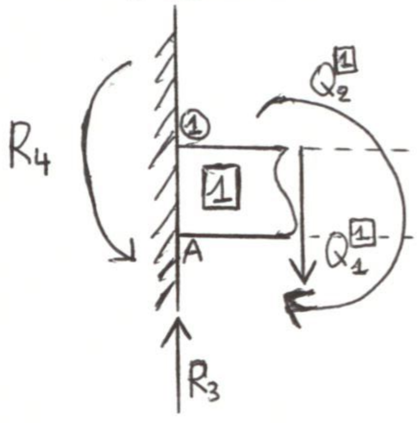
Assembling [S] & $\{P_f\}$ using Rigorous Method

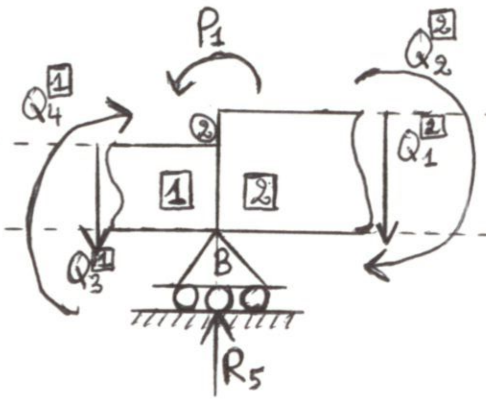
Joint ① FBD



$+\uparrow\sum F_y=0;\ \boxed{R_3=Q_1^{\boxed{1}}}$

$+\circlearrowleft\sum M_A=0;\ \boxed{R_4=Q_2^{\boxed{1}}}$

Joint ② FBD



$+\uparrow\sum F_y=0;\ \boxed{R_5=Q_3^{\boxed{1}}+Q_1^{\boxed{2}}}$

$+\circlearrowleft\sum M_B=0;$

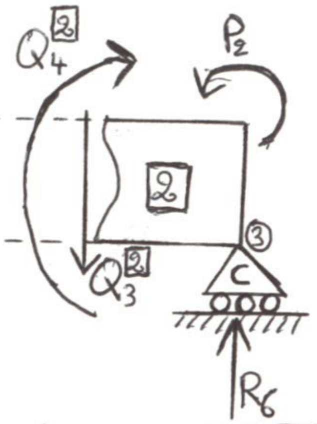
$\boxed{P_1=Q_4^{\boxed{1}}+Q_2^{\boxed{2}}}$

$$=\left(Q_{f4}^{\boxed{1}}+k_{41}^{\boxed{1}}\cancelto{0}{u_1^{\boxed{1}}}+k_{42}^{\boxed{1}}\cancelto{0}{u_2^{\boxed{1}}}+k_{43}^{\boxed{1}}\cancelto{0}{u_3^{\boxed{1}}}+k_{44}^{\boxed{1}}\cancelto{d_1}{u_4^{\boxed{1}}}\right)+\left(Q_{f2}^{\boxed{2}}+k_{21}^{\boxed{2}}\cancelto{0}{u_1^{\boxed{2}}}+k_{22}^{\boxed{2}}\cancelto{d_1}{u_2^{\boxed{2}}}+k_{23}^{\boxed{2}}\cancelto{0}{u_3^{\boxed{2}}}+k_{24}^{\boxed{2}}\cancelto{d_2}{u_4^{\boxed{2}}}\right)$$

$$P_1=\left(Q_{f4}^{\boxed{1}}+Q_{f2}^{\boxed{2}}\right)+\left(k_{44}^{\boxed{1}}+k_{22}^{\boxed{2}}\right)d_1+\left(k_{24}^{\boxed{2}}\right)d_2$$

$$\boxed{P_1=\underbrace{\left(Q_{f4}^{\boxed{1}}+Q_{f2}^{\boxed{2}}\right)}_{P_{f1}}+\underbrace{\left(k_{44}^{\boxed{1}}+k_{22}^{\boxed{2}}\right)}_{S_{11}}d_1+\underbrace{\left(k_{24}^{\boxed{2}}\right)}_{S_{12}}d_2}$$

Joint ③ FBD



$+\uparrow\sum F_y=0;\ \boxed{R_6=Q_3^{\boxed{2}}}$

$\sum M_C=0;$

$\boxed{P_2=Q_4^{\boxed{2}}}$

$$P_2=\left(Q_{f4}^{\boxed{2}}+k_{41}^{\boxed{2}}\cancelto{0}{u_1^{\boxed{2}}}+k_{42}^{\boxed{2}}\cancelto{d_1}{u_2^{\boxed{2}}}+k_{43}^{\boxed{2}}\cancelto{0}{u_3^{\boxed{2}}}+k_{44}^{\boxed{2}}\cancelto{d_2}{u_4^{\boxed{2}}}\right)$$

$$\boxed{P_2=\underbrace{\left(Q_{f4}^{\boxed{2}}\right)}_{P_{f2}}+\underbrace{\left(k_{42}^{\boxed{2}}\right)}_{S_{21}}d_1+\underbrace{\left(k_{44}^{\boxed{2}}\right)}_{S_{22}}d_2}$$

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Solve for displacement, $\{d\}$

$$\{P\} = \{P_f\} + [S]\{d\} \quad ; \quad \{P\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{Bmatrix} 10\ \text{kips-}\cancel{\text{ft}}\left(\frac{12\,\text{in}}{1\,\cancel{\text{ft}}}\right) \\ 0 \end{Bmatrix} = \begin{Bmatrix} 120\ \text{kips-in} \\ 0 \end{Bmatrix}$$

$$\{d\} = \left(\{P\} - \{P_f\}\right) * [S]^{-1} \quad ; \quad \{P_f\} = \begin{Bmatrix} P_{f1} \\ P_{f2} \end{Bmatrix} = \begin{Bmatrix} -8.33\ \text{kips-}\cancel{\text{ft}}\left(\frac{12\,\text{in}}{1\,\cancel{\text{ft}}}\right) \\ 0 \end{Bmatrix} = \begin{Bmatrix} -99.96\ \text{kips-in} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \left(\begin{Bmatrix} 120\ \text{kips-in} \\ 0 \end{Bmatrix} - \begin{Bmatrix} -99.96\ \text{kips-in} \\ 0 \end{Bmatrix} \right) * \begin{bmatrix} 966{,}666.67 & 290{,}000 \\ 290{,}000 & 580{,}000 \end{bmatrix}^{-1} \frac{\text{kips}}{\text{in}}$$

$$= \begin{Bmatrix} 219.96\ \text{kips-in} \\ 0 \end{Bmatrix} * \begin{bmatrix} 1.21704 \times 10^{-6} & -6.08519 \times 10^{-7} \\ -6.08519 \times 10^{-7} & 2.0284 \times 10^{-6} \end{bmatrix}$$

$$\boxed{\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \begin{Bmatrix} 2.677 * 10^{-4} \\ -1.3385 \times 10^{-4} \end{Bmatrix} \text{rad}} \rightarrow \text{displacements}$$

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Member Forces

$$\{Q\}^{\boxed{1}} = \{Q_f\}^{\boxed{1}} + [k]^{\boxed{1}}\{u\}^{\boxed{1}}$$

$$= \begin{Bmatrix} 10 \text{ kips} \\ 99.96 \text{ kips-in} \\ 10 \text{ kips} \\ -99.96 \text{ kips-in} \end{Bmatrix} + \begin{bmatrix} 322.22 & 9,666.67 & -322.22 & 9,666.67 \\ 9,666.67 & 386,666.67 & -9,666.67 & 193,333.33 \\ -322.22 & -9,666.67 & 322.22 & -9,666.67 \\ 9,666.67 & 193,333.33 & -9,666.67 & 386,666.67 \end{bmatrix} \underbrace{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.6 \times 10^{-4} \text{(rad)} \end{Bmatrix}}_{\text{d}_1}$$

$$\boxed{\{Q\}^{\boxed{1}} = \begin{Bmatrix} 12.59 \text{ kips} \\ 151.7153324 \text{ kips-in} \\ 7.412 \text{ kips} \\ 3.550667559 \text{ kips-in} \end{Bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 1 \end{matrix}}$$

$Q_3^{\boxed{1}}$

$$\{Q\}^{\boxed{2}} = \{Q_f\}^{\boxed{2}} + [k]^{\boxed{2}}\{u\}^{\boxed{2}}$$

$$= \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{bmatrix} 483.33 & 14,500 & -483.33 & 14,500 \\ 14,500 & 580,000 & -14,500 & 290,000 \\ -483.33 & -14,500 & 483.33 & -14,500 \\ 14,500 & 290,000 & -14,500 & 580,000 \end{bmatrix} \begin{Bmatrix} 0 \\ \overset{d_1}{2.677 * 10^{-4} \text{ rad}} \\ 0 \\ \underset{d_2}{-1.5385 * 10^{-4} \text{ rad}} \end{Bmatrix}$$

$$\boxed{\{Q\}^{\boxed{2}} = \left\{\begin{array}{ll} 1.940825 & \text{kips} \\ 116.4495 & \text{kips-in} \\ -1.940825 & \text{kips} \\ 0 & \text{kips-in} \end{array}\right\} \begin{array}{l} 5 \\ 1 \\ 6 \\ 2 \end{array}}$$

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Reactions from member forces (using Code Number Method)

$$\{Q\}^{\boxed{1}} = \begin{Bmatrix} 12.59 \text{ kips} \\ 151.7153324 \text{ kips-in} \\ 7.412 \text{ kips} \\ 3.550667559 \text{ kips-in} \end{Bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 1 \end{matrix}$$

$$\{Q\}^{\boxed{2}} = \begin{Bmatrix} 1.940825 \text{ kips} \\ 116.4495 \text{ kips-in} \\ -1.940825 \text{ kips} \\ 0 \text{ kips-in} \end{Bmatrix} \begin{matrix} 5 \\ 1 \\ 6 \\ 2 \end{matrix}$$

$$\{R\} = \begin{Bmatrix} R_3 \\ R_4 \\ R_5 \\ R_6 \end{Bmatrix} = \begin{Bmatrix} 12.59 \text{ kips} \\ 151.72 \text{ kips-}\cancel{\text{in}}\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right) = 12.6 \text{ kips-ft} \\ (7.412 \text{ kips}) + (1.941 \text{ kips}) = 9.353 \text{ kips} \\ -1.941 \text{ kips} \end{Bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

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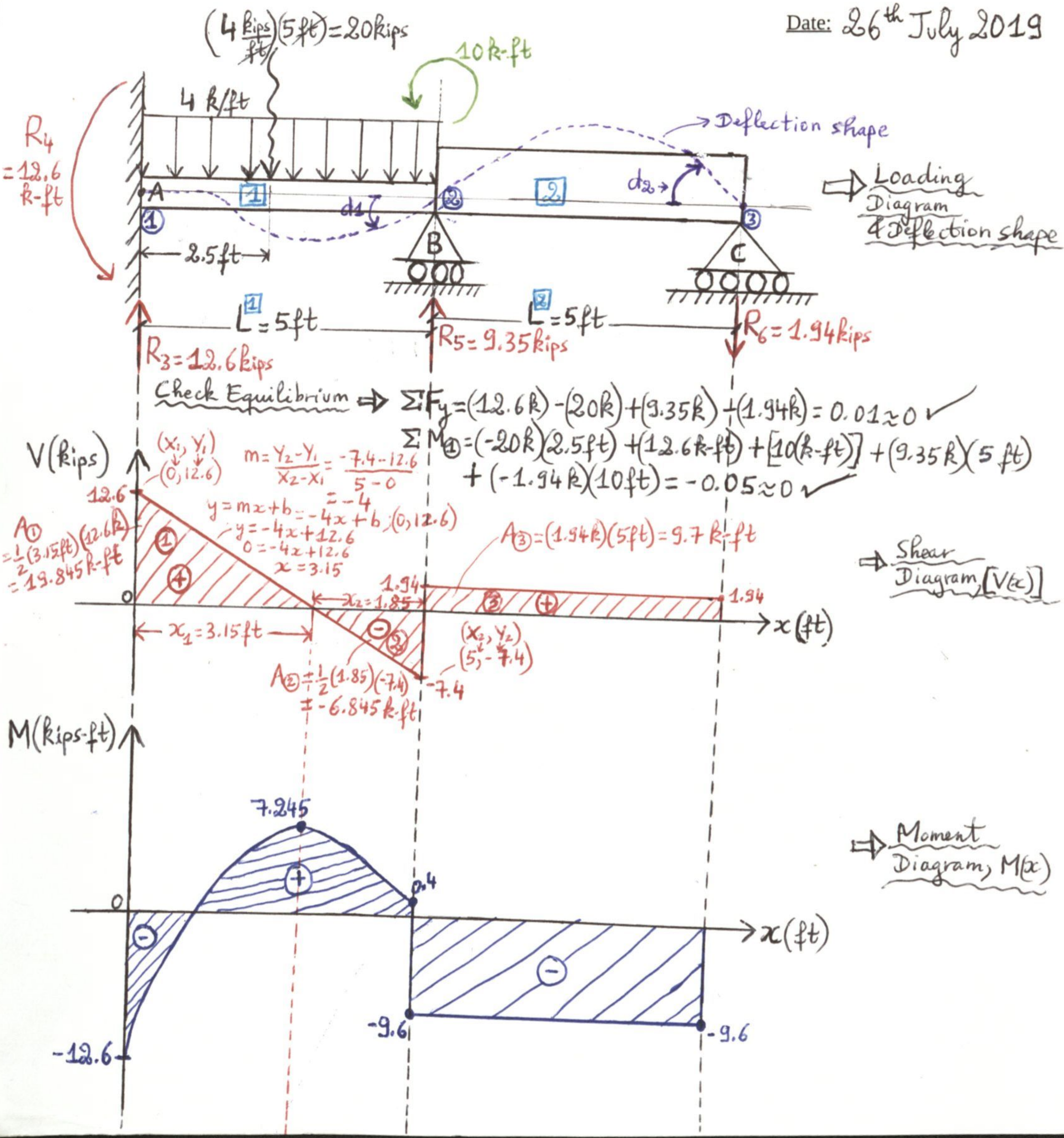

Check Equilibrium ⇒ $\Sigma F_y = (12.6k) - (20k) + (9.35k) + (1.94k) = 0.01 \approx 0$ ✓

$\Sigma M_① = (-20k)(2.5ft) + (12.6k\text{-}ft) + [10(k\text{-}ft)] + (9.35k)(5ft)$
$+ (-1.94k)(10ft) = -0.05 \approx 0$ ✓