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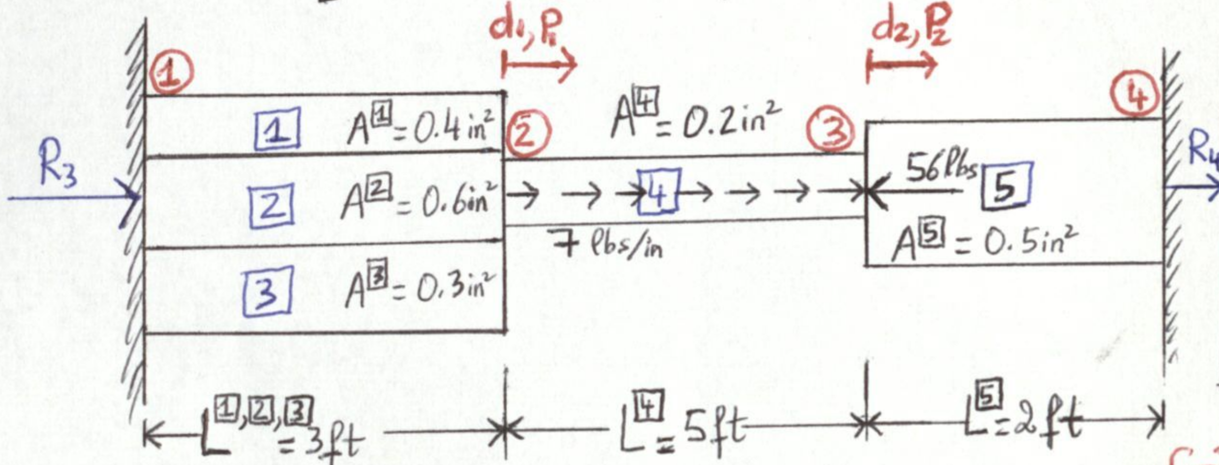
Date: 11th September 2018

Axial Problem Example 2

$E^{\boxed{1},\boxed{2},\boxed{3},\boxed{4},\boxed{5}} = 30\times10^6 psi$



Member	Code #
$\boxed{1}$	[3 1]
$\boxed{2}$	[3 1]
$\boxed{3}$	[3 1]
$\boxed{4}$	[1 2]
$\boxed{5}$	[2 4]

$$\{P\} = \begin{Bmatrix} 0 \\ -56\ lbs \end{Bmatrix}$$

$$[k]^{\boxed{1}} = \frac{EA^{\boxed{1}}}{L^{\boxed{1}}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{(30\times10^6 psi)(0.4\,in^2)}{(3\cancel{ft})\left(\frac{12\,in}{1\cancel{ft}}\right)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{array}{c} \begin{array}{cc} 3 & 1 \end{array} \\ \begin{bmatrix}333,333.3 & -333,333.3\\ -333,333.33 & 333,333.3\end{bmatrix}\end{array}\begin{array}{c}3\\1\end{array}\ \frac{lbs}{in}$$

$$[k]^{\boxed{2}} = \frac{EA^{\boxed{2}}}{L^{\boxed{2}}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{(30\times10^6 psi)(0.6\,in^2)}{(3\cancel{ft})\left(\frac{12\,in}{1\cancel{ft}}\right)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{array}{c} \begin{array}{cc} 3 & 1 \end{array} \\ \begin{bmatrix}5\times10^5 & -5\times10^5\\ -5\times10^5 & 5\times10^5\end{bmatrix}\end{array}\begin{array}{c}3\\1\end{array}\ \frac{lbs}{in}$$

$$[k]^{\boxed{3}} = \frac{EA^{\boxed{3}}}{L^{\boxed{3}}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{(30\times10^6 psi)(0.3\,in^2)}{(3\cancel{ft})\left(\frac{12\,in}{1\cancel{ft}}\right)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{array}{c} \begin{array}{cc} 3 & 1 \end{array} \\ \begin{bmatrix}25\times10^4 & -25\times10^4\\ -25\times10^4 & 25\times10^4\end{bmatrix}\end{array}\begin{array}{c}3\\1\end{array}\ \frac{lbs}{in}$$

$$[k]^{\boxed{4}} = \frac{EA^{\boxed{4}}}{L^{\boxed{4}}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{(30\times10^6 psi)(0.2\,in^2)}{(5\cancel{ft})\left(\frac{12\,in}{1\cancel{ft}}\right)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix}1\times10^5 & -1\times10^5\\ -1\times10^5 & 1\times10^5\end{bmatrix}\end{array}\begin{array}{c}1\\2\end{array}\ \frac{lbs}{in}$$

$$[k]^{\boxed{5}} = \frac{EA^{\boxed{5}}}{L^{\boxed{5}}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{(30\times10^6 psi)(0.5\,in^2)}{(2\cancel{ft})\left(\frac{12\,in}{1\cancel{ft}}\right)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{array}{c} \begin{array}{cc} 2 & 4 \end{array} \\ \begin{bmatrix}62.5\times10^4 & -62.5\times10^4\\ -62.5\times10^4 & 62.5\times10^4\end{bmatrix}\end{array}\begin{array}{c}2\\4\end{array}\ \frac{lbs}{in}$$

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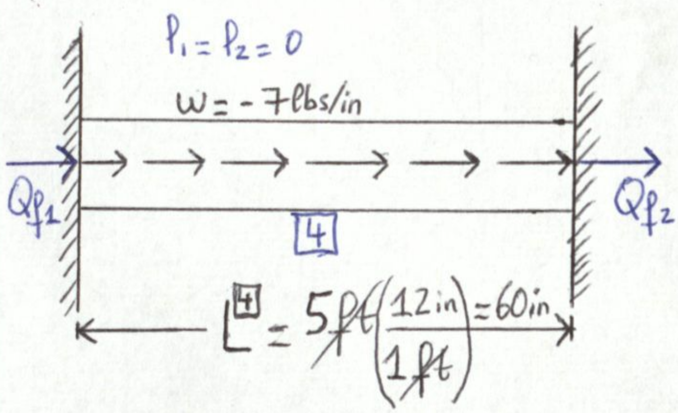
Axial Problem Example 2 continued...

$$[S] = \begin{array}{c|cc|c} & 1 & 2 & \\ \hline & (333{,}333)+(5\times10^5)+(25\times10^4)+(1\times10^5) & (-1\times10^5) & 1 \\ & (-1\times10^5) & (1\times10^5)+(62.5\times10^4) & 2 \end{array} = \begin{array}{cc} 1 & 2 \\ \left[\begin{matrix} 11.8\times10^5 & -1\times10^5 \\ -1\times10^5 & 7.25\times10^5 \end{matrix}\right] \end{array} \begin{matrix} 1 \\ 2 \end{matrix} \ \frac{lbs}{in}$$

No Fixed-End Forces for Member $\boxed{1}$, $\boxed{2}$, $\boxed{3}$ $\rightarrow \{Q_f\}^{\boxed{1},\boxed{2},\boxed{3}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \begin{matrix} 3 \\ 1 \end{matrix}$

Fixed-End Forces Member $\boxed{4}$



$$Q_{f1}^{\boxed{4}} = \frac{w}{2L^{\boxed{4}}}(L-\ell_1-\ell_2)(L-\ell_1+\ell_2)$$

$$= \frac{(-7\,lbs/in)}{2(60in)}[60in-0-0][60in-0+0]$$

$$Q_{f1}^{\boxed{4}} = -210\,lbs$$

$$Q_{f2}^{\boxed{4}} = \frac{w}{2L^{\boxed{4}}}(L-\ell_1-\ell_2)(L+\ell_1-\ell_2)$$

$$= \frac{(-7\,lbs/in)}{2(60in)}[60-0-0][60+0-0]$$

$$Q_{f2}^{\boxed{4}} = -210\,lbs$$

$$\{Q_f\}^{\boxed{4}} = \begin{Bmatrix} -210\,lbs \\ -210\,lbs \end{Bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

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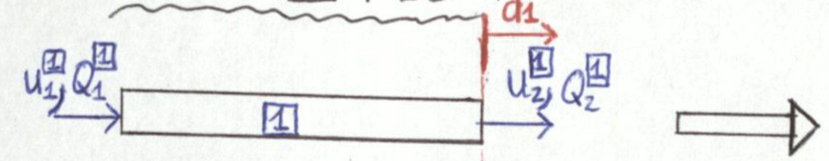
Axial Problem Example 2 continued...

No Fixed-End Force for Member $\boxed{5}$ $\rightarrow$ $\{Q_f\}^{\boxed{5}} = \begin{Bmatrix} 0 \text{ lbs} \\ 0 \text{ lbs} \end{Bmatrix} \begin{matrix} 2 \\ 4 \end{matrix}$

$$\{P_f\} = \begin{Bmatrix} -210 \text{ lbs} \\ -210 \text{ lbs} \end{Bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Member $\boxed{1}$ FBD

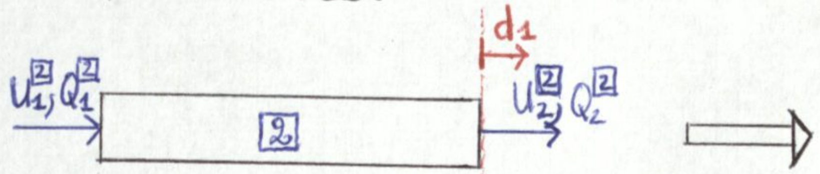


Compatibility

$u_1^{\boxed{1}} = 0$

$u_2^{\boxed{1}} = d_1$

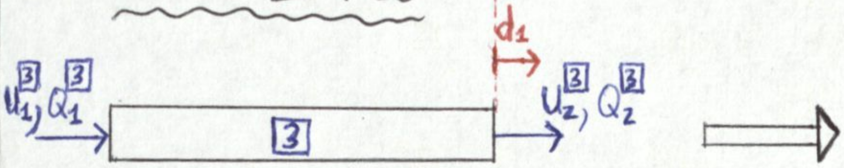
Member $\boxed{2}$ FBD



$u_1^{\boxed{2}} = 0$

$u_2^{\boxed{2}} = d_1$

Member $\boxed{3}$ FBD



$u_1^{\boxed{3}} = 0$

$u_2^{\boxed{3}} = d_1$

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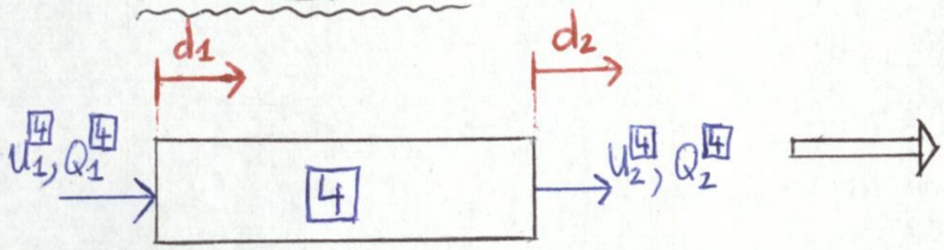
Axial Problem Example 2 continued...

Member $\boxed{4}$ FBD

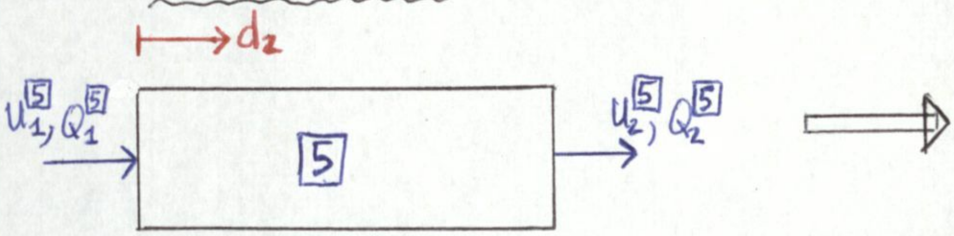


Compatibility

$u_1^{\boxed{4}} = d_1$

$u_2^{\boxed{4}} = d_2$

Member $\boxed{5}$ FBD



$u_1^{\boxed{5}} = d_2$

$u_2^{\boxed{5}} = 0$

Solve for $\{d\}$

$$\{P\} = \{P_f\} + [S]\{d\}$$

$$\{d\} = \frac{\{P\} - \{P_f\}}{[S]} = \left[\{P\} - \{P_f\}\right] * [S]^{-1}$$

$$= \left[\begin{Bmatrix} 0 \\ -56 \end{Bmatrix} - \begin{Bmatrix} -210 \\ -210 \end{Bmatrix}\right] * \begin{bmatrix} 8.574 \times 10^{-7} & 1.18273 \times 10^{-7} \\ 1.18273 \times 10^{-7} & 0.000001 \end{bmatrix}$$

$$= \begin{Bmatrix} 210 \\ 154 \end{Bmatrix} * \begin{bmatrix} 8.574 \times 10^{-7} & 1.18273 \times 10^{-7} \\ 1.18273 \times 10^{-7} & 0.000001 \end{bmatrix}$$

$$\boxed{\{d\} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \begin{Bmatrix} 0.000198 \\ 0.00024 \end{Bmatrix} \text{in}}$$

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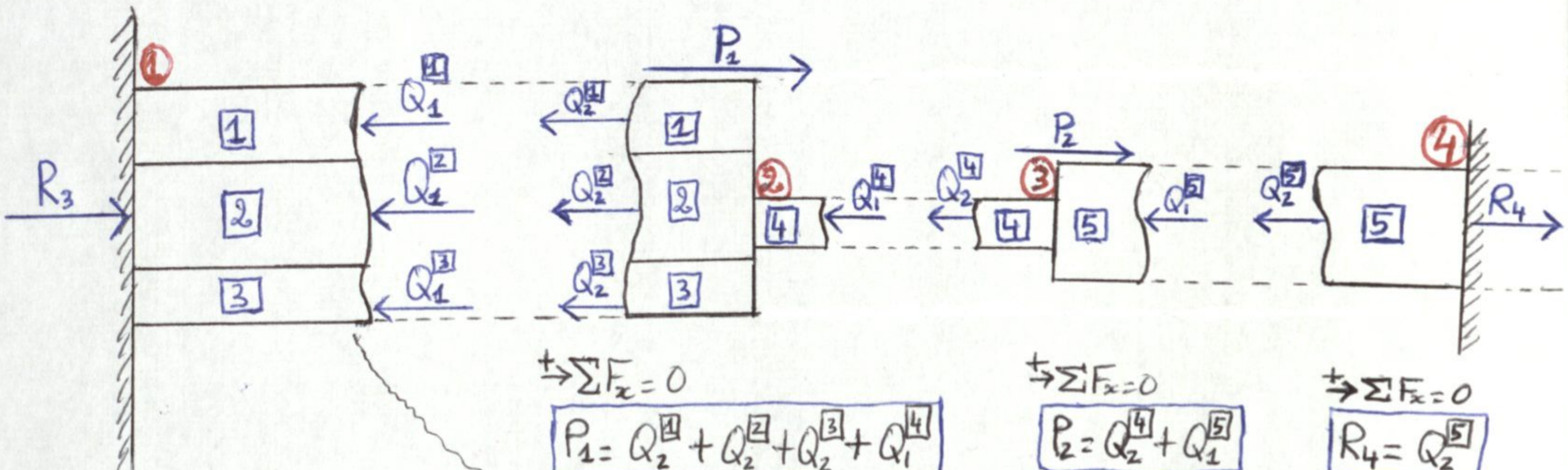
Axial Problem Example 2 continued...

Joint 1 FBD   Joint 2 FBD   Joint 3 FBD   Joint 4 FBD



$\overset{+}{\rightarrow}\sum F_x = 0$

$$\boxed{P_1 = Q_2^{\boxed{1}} + Q_2^{\boxed{2}} + Q_2^{\boxed{3}} + Q_1^{\boxed{4}}}$$

$\overset{+}{\rightarrow}\sum F_x = 0$

$$\boxed{P_2 = Q_2^{\boxed{4}} + Q_1^{\boxed{5}}}$$

$\overset{+}{\rightarrow}\sum F_x = 0$

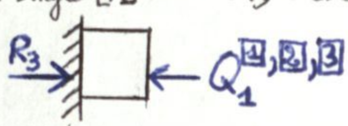
$$\boxed{R_4 = Q_2^{\boxed{5}}}$$

$\overset{+}{\rightarrow}\sum F_x = 0;$

$$\boxed{R_3 = Q_1^{\boxed{1}} + Q_1^{\boxed{2}} + Q_1^{\boxed{3}}}$$

Or you could combine $\boxed{1}$, $\boxed{2}$ & $\boxed{3}$ to form a single $[k]$ matrix; then you would have only a single Q.

$$[k] = \frac{E\left(A^{\boxed{1}} + A^{\boxed{2}} + A^{\boxed{3}}\right)}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



Member Forces

$$\{Q_1\}^{\boxed{1}} = Qf_1^{\boxed{1}} + k_{11}^{\boxed{1}}\cancelto{0}{u_1^{\boxed{1}}} + k_{12}^{\boxed{1}}\cancelto{d_1}{u_2^{\boxed{1}}} = (0) + (333{,}333)(0) + (-333{,}333)(0.000198\,in) = -65.9\ lbs$$

$$\{Q_2\}^{\boxed{1}} = Qf_2^{\boxed{1}} + k_{21}^{\boxed{1}}\cancelto{0}{u_1^{\boxed{1}}} + k_{22}^{\boxed{1}}\cancelto{d_1}{u_2^{\boxed{1}}} = (0) + (-333{,}333)(0) + (333{,}333)(0.000198\,in) = 65.9\ lbs$$

$$\{Q_1\}^{\boxed{2}} = Qf_1^{\boxed{2}} + k_{11}^{\boxed{2}}\cancelto{0}{u_1^{\boxed{2}}} + k_{12}^{\boxed{2}}\cancelto{d_1}{u_2^{\boxed{2}}} = (0) + (5\times10^5)(0) + (-5\times10^5)(0.000198\,in) = -99\ lbs$$

$$\{Q_2\}^{\boxed{2}} = Qf_2^{\boxed{2}} + k_{21}^{\boxed{2}}\cancelto{0}{u_1^{\boxed{2}}} + k_{22}^{\boxed{2}}\cancelto{d_1}{u_2^{\boxed{2}}} = (0) + (-5\times10^5)(0) + (5\times10^5)(0.000198\,in) = 99\ lbs$$

$$\{Q_1\}^{\boxed{3}} = Qf_1^{\boxed{3}} + k_{11}^{\boxed{3}}\cancelto{0}{u_1^{\boxed{3}}} + k_{12}^{\boxed{3}}\cancelto{d_1}{u_2^{\boxed{3}}} = (0) + (25\times10^4)(0) + (-25\times10^4)(0.000198\,in) = -49.5\ lbs$$

$$\{Q_2\}^{\boxed{3}} = Qf_2^{\boxed{3}} + k_{21}^{\boxed{3}}\cancelto{0}{u_1^{\boxed{3}}} + k_{22}^{\boxed{3}}\cancelto{d_1}{u_2^{\boxed{3}}} = (0) + (-25\times10^4)(0) + (25\times10^4)(0.000198\,in) = 49.5\ lbs$$

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Axial Problem Example 2 continued...

$$\{Q_1\}^{\boxed{4}} = Qf_1^{\boxed{4}} + k_{11}^{\boxed{4}}\overset{d_1}{\cancel{u_1^{\boxed{4}}}} + k_{12}^{\boxed{4}}\overset{d_2}{\cancel{u_2^{\boxed{4}}}} = (-210) + (1\times10^5)(0.000198) + (-1\times10^5)(0.00024) = -214.2\text{ lbs}$$

$$\{Q_2\}^{\boxed{4}} = Qf_2^{\boxed{4}} + k_{21}^{\boxed{4}}\overset{d_1}{\cancel{u_1^{\boxed{4}}}} + k_{22}^{\boxed{4}}\overset{d_2}{\cancel{u_2^{\boxed{4}}}} = (-210) + (-1\times10^5)(0.000198) + (1\times10^5)(0.00024) = -205.8\text{ lbs}$$

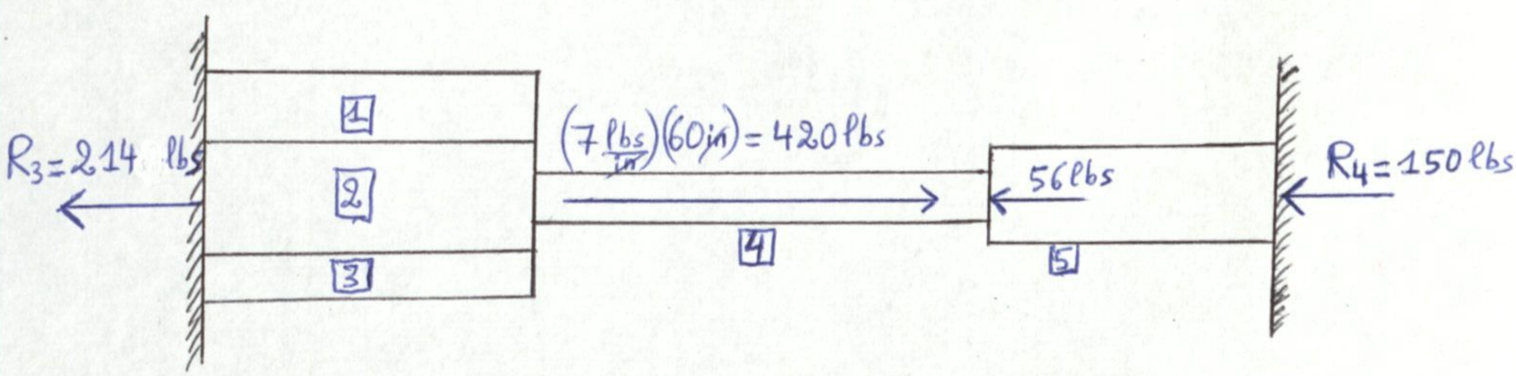
$$\{Q_1\}^{\boxed{5}} = Qf_1^{\boxed{5}} + k_{11}^{\boxed{5}}\overset{d_2}{\cancel{u_1^{\boxed{5}}}} + k_{12}^{\boxed{5}}\overset{0}{\cancel{u_2^{\boxed{5}}}} = (0) + (62.5\times10^4)(0.00024) + (-62.5\times10^4)(0) = 150\text{ lbs}$$

$$\{Q_2\}^{\boxed{5}} = Qf_2^{\boxed{5}} + k_{21}^{\boxed{5}}\overset{d_2}{\cancel{u_1^{\boxed{5}}}} + k_{22}^{\boxed{5}}\overset{0}{\cancel{u_2^{\boxed{5}}}} = (0) + (-62.5\times10^4)(0.00024) + (62.5\times10^4)(0) = -150\text{ lbs}$$

$$R_3 = Q_1^{\boxed{1}} + Q_1^{\boxed{2}} + Q_1^{\boxed{3}} = (-65.9) + (-99) + (-49.5) = -214\text{ lbs}$$

$$R_4 = Q_2^{\boxed{5}} = -150\text{ lbs}$$

Check Equilibrium



$$\overset{+}{\rightarrow}\sum F_x \overset{?}{=} 0$$

$$(-214\text{ lbs}) + (420\text{ lbs}) - (56\text{ lbs}) - (150\text{ lbs}) \overset{\checkmark}{=} 0$$

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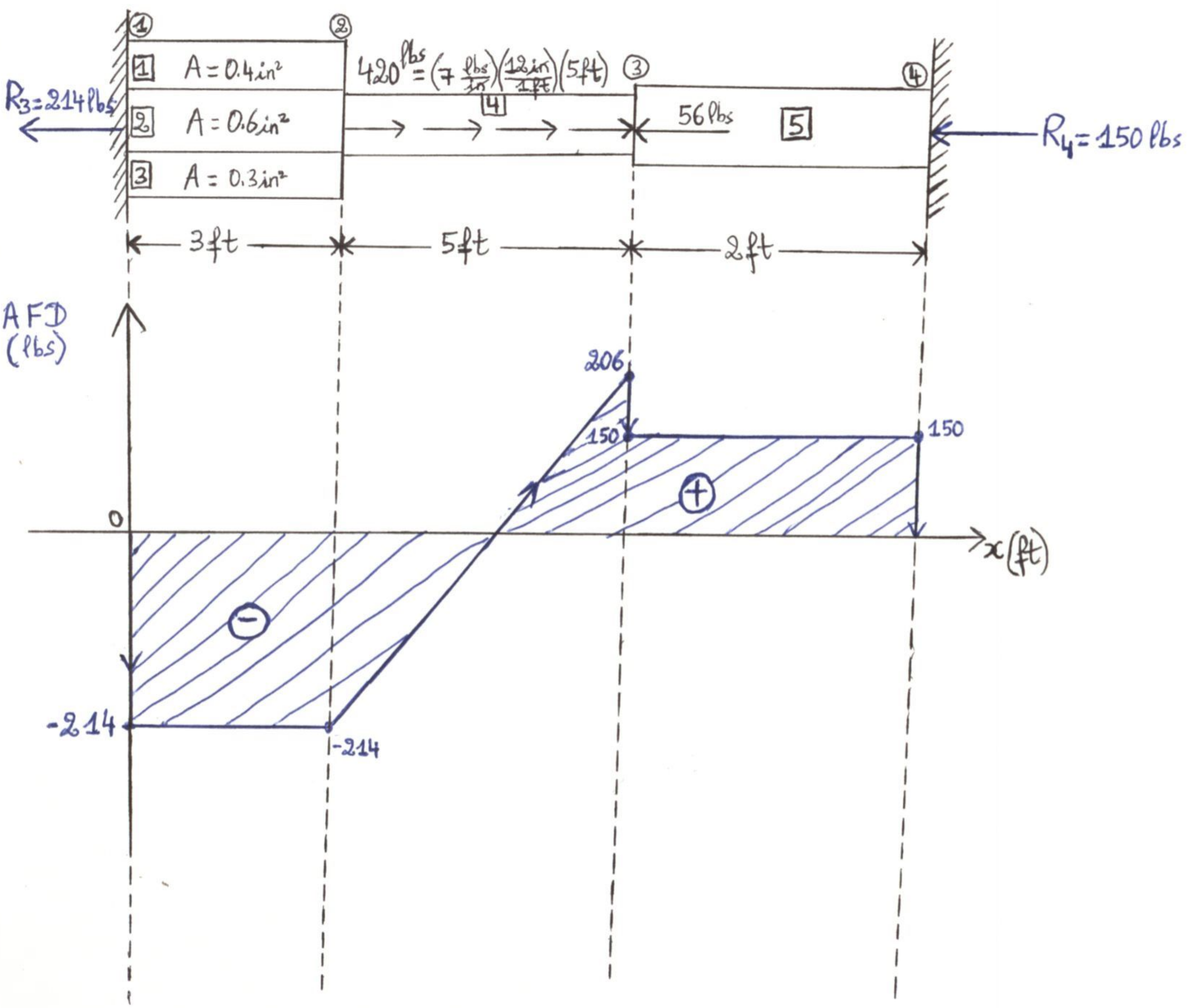
Axial Force Diagram (AFD)



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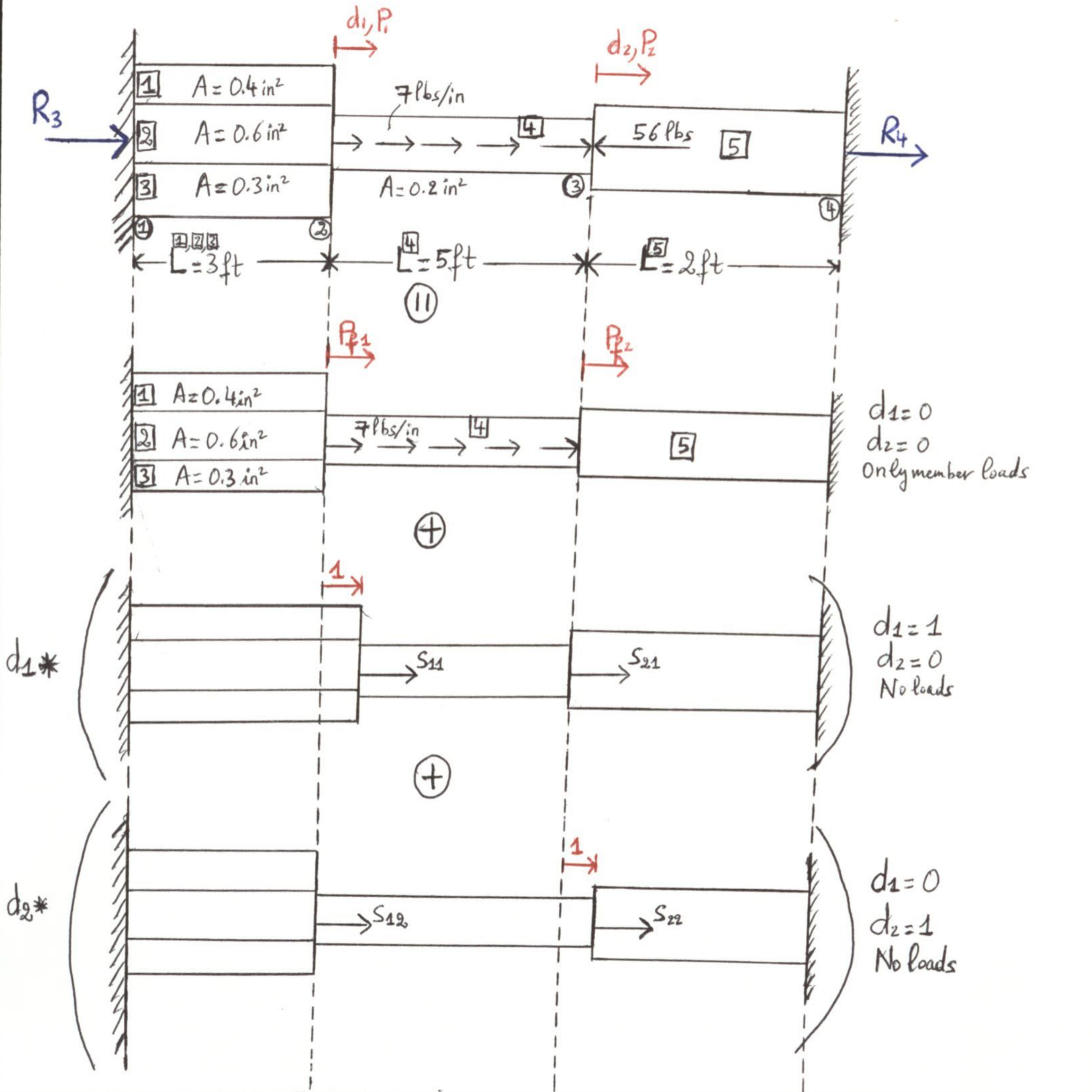
Structural Level Superposition (SLS)



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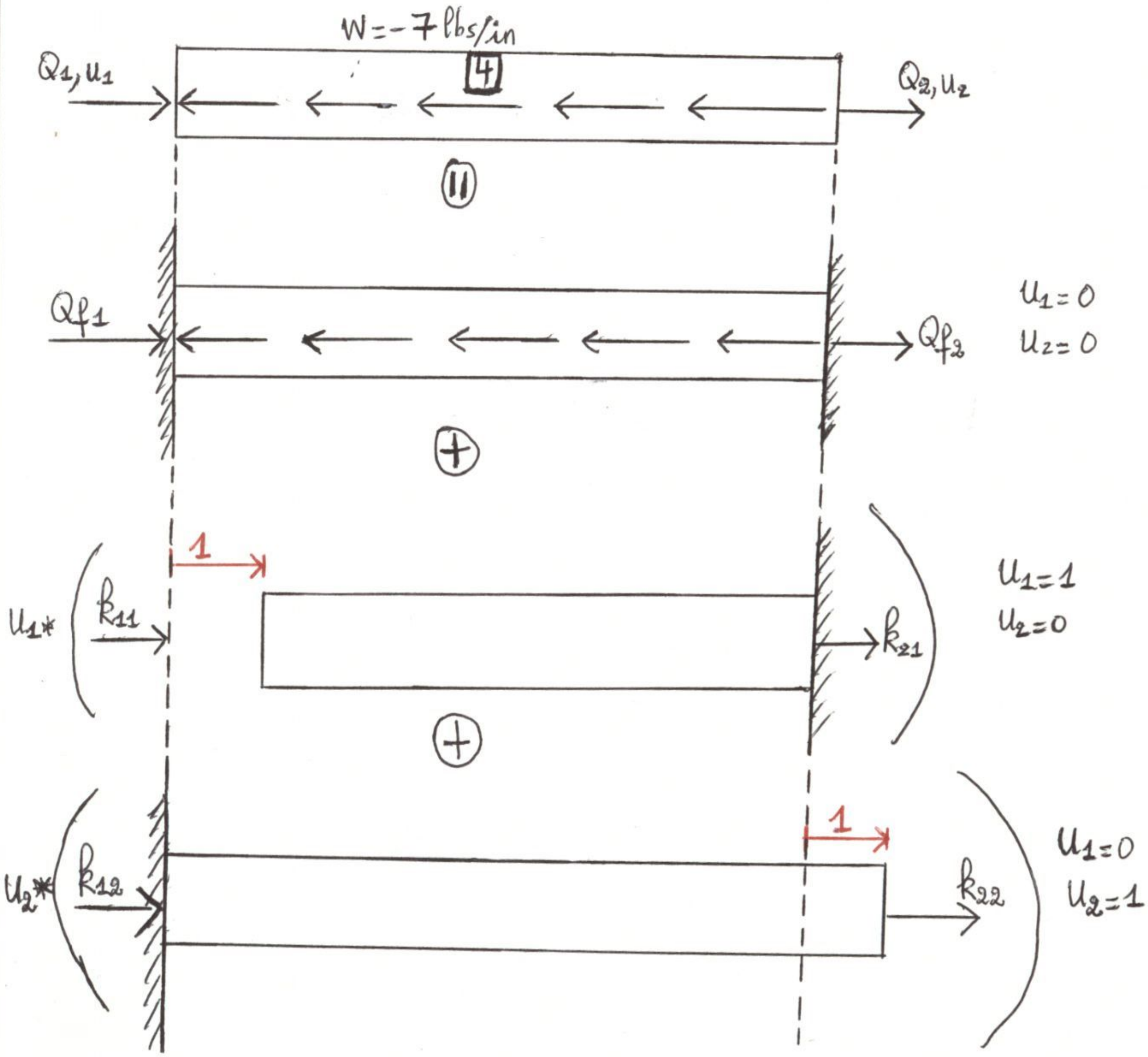
Member Level Superposition (MLS) for member 4



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Axial Problem Example 2 continued...

Steps

(1) Draw FBD.

(2) Label all joints in circle ○ & the members in square □.

(3) Label all Degrees Of Freedom (DOF_s).

(4) Write down the code number for each member.

(5) Compute the local stiffness, $[k_{ij}]$, for each member & assemble the global stiffness, $[S]$ using code number method or rigorous method.

(6) Calculate the fixed-end forces for each member using tabulated equations & assemble $\{P_f\}$ using code number method.

(7) Draw a member FBD for each member and write compatibility conditions.

(8) Draw a joint FBD for each joint and write down the equilibrium equations to find P & R in terms of Q.

(9) Assemble $\{P\}$ using code number method.

(10) Compute the displacement, $\{d\} = \left[\{P\} - \{P_f\}\right] * [S]^{-1}$

(11) Compute the member forces, Q, for each member & using the equations found in part (8), calculate the reactions.

(12) Draw the entire FBD for the whole structure with all forces and reactions and check equilibrium, ΣF_x has to be equal or very very close to zero, $\Sigma F_x \simeq 0$, if it is not, then there is a problem, go back and see if you made some mistakes.

(13) Draw Axial Force Diagram (AFD).

(14) Draw MLS & SLS (optional).