DTY TUTORING, inc.

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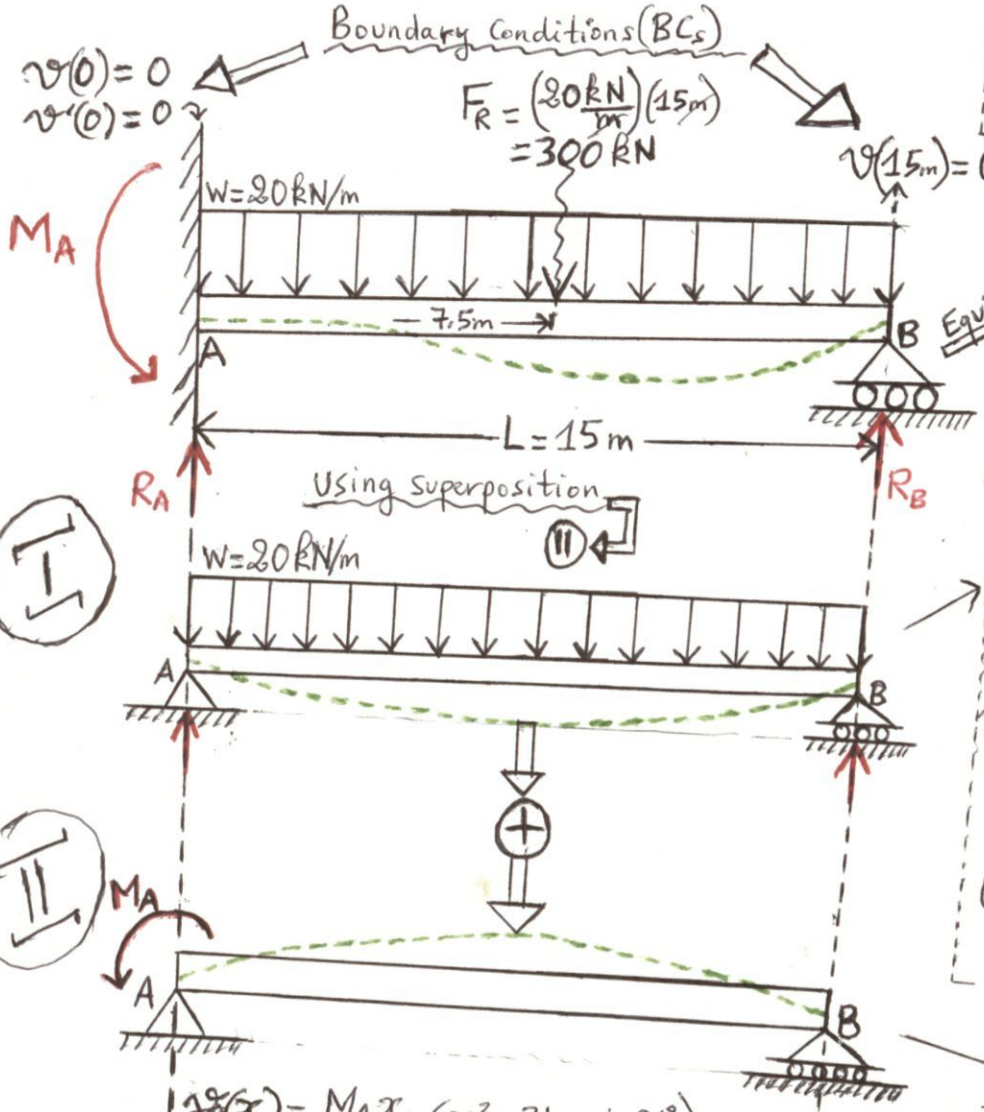
Date: 17th September 2019



$+\circlearrowleft \sum M_A = 0;\ (-300\,kN)(7.5m) + (R_B)(15m) + M_A = 0$ ① $\Rightarrow M_A = 2{,}250 - 15R_B$

$+\uparrow \sum F_y = 0;\ R_A + R_B = 300\,kN$

② $R_A = 300 - R_B$

Equilibrium

Using tables

(a) ↓

$$v(x)_{(II)} = \frac{-wx}{24EI}\left[x^3 - 2Lx^2 + L^3\right]$$

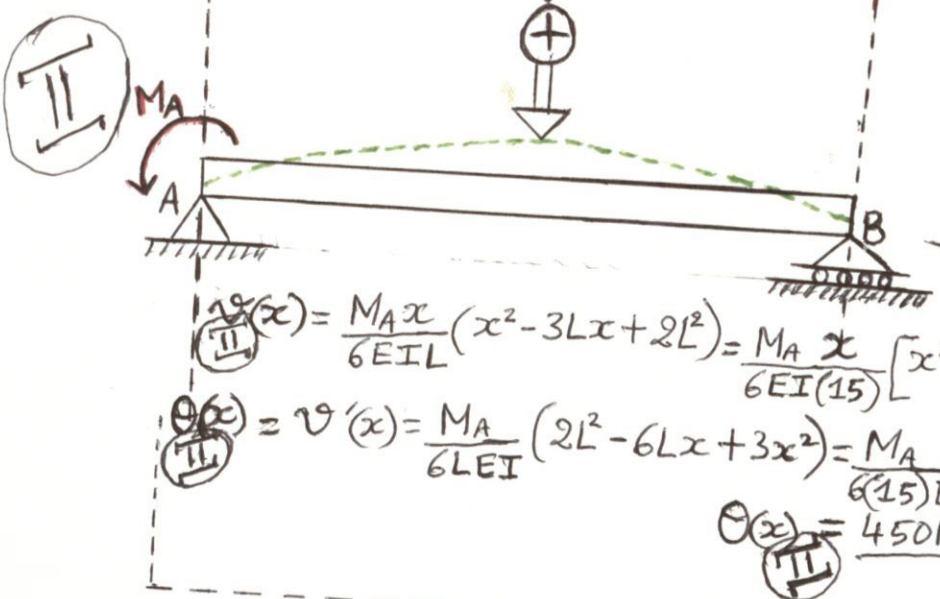
$$= \frac{-(20)}{24EI}x\left[x^3 - 2(15)x^2 + (15)^3\right]$$

$$v(x)_{(II)} = \frac{-20x^4 + 600x^3 - 67{,}500x}{24EI}$$

$$\theta(x)_{(I)} = v'(x) = -\frac{w}{24EI}\left(L^3 - 6Lx^2 + 4x^3\right)$$

$$= \frac{-20}{24EI}\left[(15)^3 - 6(15)x^2 + 4x^3\right]$$

$$\theta(x)_{(I)} = \frac{-67{,}500 + 1{,}800x^2 - 80x^3}{24EI}$$



$$v(x)_{(II)} = \frac{M_A x}{6EIL}\left(x^2 - 3Lx + 2L^2\right) = \frac{M_A x}{6EI(15)}\left[x^2 - 3(15)x + 2(15)^2\right] = \frac{M_A x^3 - 45M_A x^2 + 450M_A x}{90EI}$$

$$\theta(x)_{(II)} = v'(x) = \frac{M_A}{6LEI}\left(2L^2 - 6Lx + 3x^2\right) = \frac{M_A}{6(15)EI}\left[2(15)^2 - 6(15)x + 3x^2\right]$$

$$\theta(x)_{(II)} = \frac{450M_A - 90M_A x + 3M_A x^2}{90EI}$$

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(a)

$$v(x) = v(x)_{\text{I}} + v(x)_{\text{II}}$$

Deflection Equation

$$= \left[\frac{-20x^4 + 600x^3 - 67{,}500x}{24EI}\right] + \left[\frac{M_A x^3 - 45M_A x^2 + 450M_A x}{90EI}\right]$$

$$v(x) = \frac{-300x^4 + (4M_A + 9{,}000)x^3 - 180M_A x^2 + (1{,}800M_A - 1{,}012{,}500)x}{360EI}$$

$$\theta(x) = v'(x) = \theta(x)_{\text{I}} + \theta(x)_{\text{II}}$$

Slope Equation

$$= \left[\frac{-67{,}500 + 1{,}800x^2 - 80x^3}{24EI}\right] + \left[\frac{450M_A - 90M_A x + 3M_A x^2}{90EI}\right]$$

$$\theta(x) = -\frac{1{,}200x^3 + (27{,}000 + 12M_A)x^2 - (360M_A)x + (1{,}800M_A - 1{,}012{,}500)}{360EI}$$

(b) Find reactions using Boundary Conditions (BCs) & Equilibrium Equations

$\theta(0) = v'(0) = 0 \Rightarrow$

$$0 = -\frac{1{,}200(0)^3 + (27{,}000 + 12M_A)(0)^2 - (360M_A)(0) + (1{,}800M_A - 1{,}012{,}500)}{360EI}$$

$1{,}800M_A = 1{,}012{,}500 \Rightarrow$

$$M_A = 1{,}012{,}500 / 1{,}800 = 562.5 \text{ kN-m}$$

From (1) $\Rightarrow M_A = 2{,}250 - 15R_B \Rightarrow$

$$R_B = \frac{2{,}250 - (562.5\text{kN-m})}{15} = 112.5 \text{ kN}$$

From (2) $\Rightarrow$

$$R_A = 300 - R_B = (300\text{kN}) - (112.5\text{kN}) = 187.5 \text{ kN}$$

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(c) Location where v_{MAX} occurs $\Rightarrow$ Set $v'(x) = 0$ to find location (x)

$$(360EI)(v'(x)) = -1{,}200x^3 + (27{,}000 + 12M_A)x^2 - (360M_A)x + (1{,}800M_A - 1{,}012{,}500)$$

$$= -1{,}200x^3 + (27{,}000 + 12(562.5))x^2 - (360(562.5))x + (1{,}800(562.5) - 1{,}012{,}500)$$

$$= -1{,}200x^3 + 33{,}750x^2 - 202{,}500x + 0$$

$$0 = -1{,}200x^3 + 33{,}750x^2 - 202{,}500x \text{ ; for } 0 < x \leq 15\text{m}$$

$x = 8.67697\text{m} \simeq 8.68\text{m}$ → Location where v_{MAX} occurs

Plug $x = 8.68$m into $v(x)$ equation in order to find v_{MAX} ; $E = 200 \times 10^6$ kPa

$$I = 600 \times 10^5\ \text{mm}^4 \left(\frac{1\ \text{m}^4}{1 \times 10^{12}\ \text{mm}^4}\right) = 600 \times 10^{-7}\ \text{m}^4$$

$$v(x) = \frac{-300x^4 + (4M_A + 9{,}000)x^3 - (180M_A)x^2 + (1{,}800M_A - 1{,}012{,}500)x}{360EI}$$

$$= \frac{-300x^4 + (4(562.5) + 9{,}000)x^3 - [180(562.5)]x^2 + [1{,}800(562.5) - 1{,}012{,}500]x}{360EI}$$

$$= \frac{-300x^4 + 11{,}250x^3 - 101{,}250x^2}{360EI}$$

$$v_{MAX} = v(8.68\text{m}) = \frac{-300(8.67697)^4 + 11{,}250(8.67697)^3 - 101{,}250(8.67697)^2}{360(200 \times 10^6\ \text{kPa})(600 \times 10^{-7}\ \text{m}^4)}$$

$$= -0.4569853\text{m}\left(\frac{1{,}000\ \text{mm}}{1\ \text{m}}\right)$$

$v_{MAX} = 457\text{mm}\downarrow$ → Maximum Deflection

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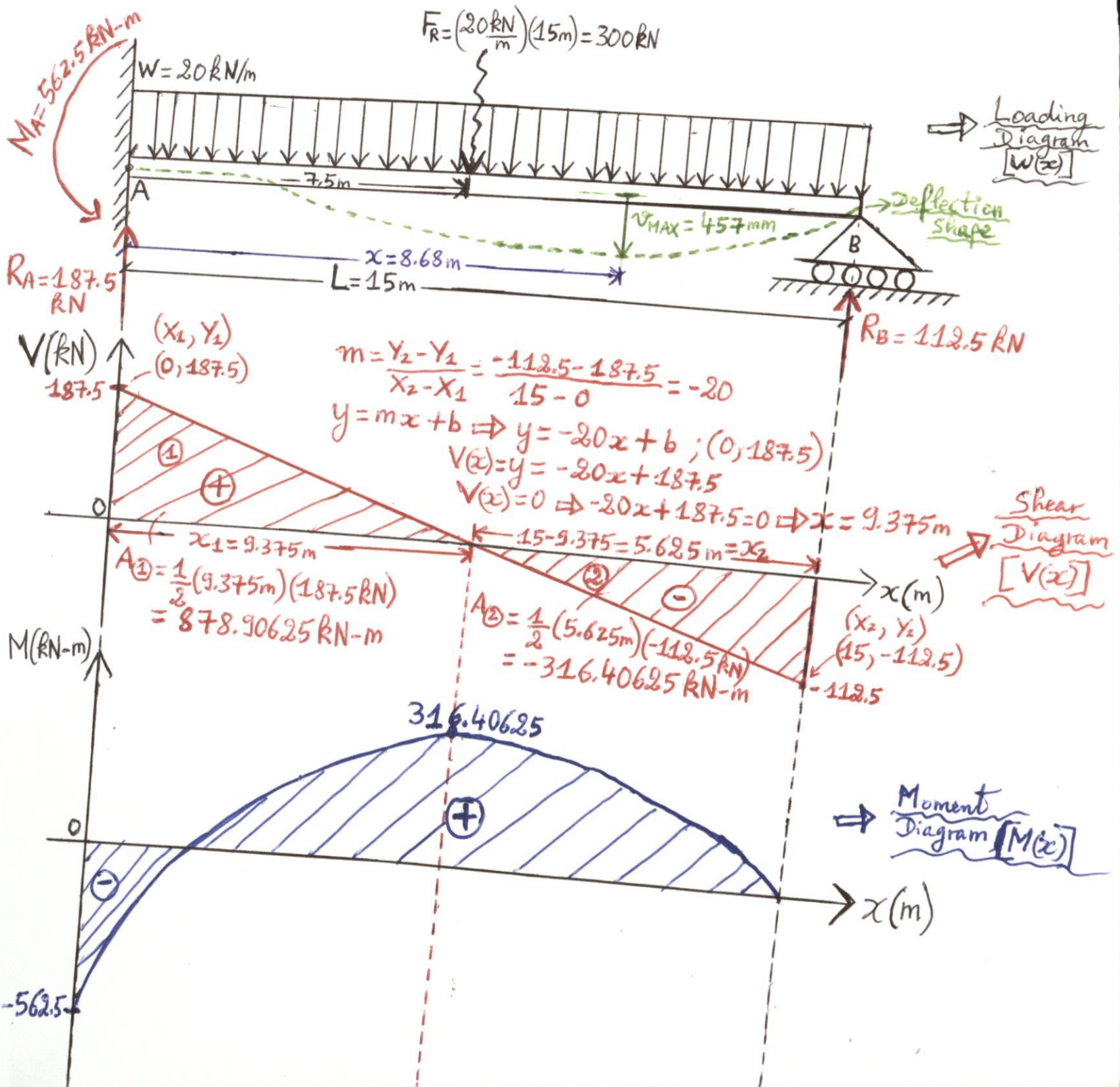
(d) Shear (V) and Moment (M) diagrams & deflection shape